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ANALYSIS PROCEDURE FOR THE INTER-SHIP CALIBRATION OF ECHO INTEGRATORS

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ABSTRACT

The relative sensitivity b of the echo integrator systems on two ships may be estimated from an inter-ship calibration. Ordinary linear regression is inappropriate since the observations from each ship are subject to error. Several alternative techniques are discussed. Formulae are presented for estimating the relative sensitivity and associated confidence intervals. The recommended estimator is that given by the method of maximum likelihood assuming the same residual (error) variance in the observations from both ships. They are applied to experimental data collected from different depths. The greater the depth, the closer are the confidence limits on b.

RESUME

La sensibilité relative b des systèmes d'écho intégration sur deux navires peut être estimée par un étalonnage réciproque des deux systèmes. La simple régression linéaire n'y convient pas puisque les observations qu'on fait sur les deux navires pourront subir également des erreurs. On discute plusieurs techniques différentes et on présente des formules pour estimer la sensibilité relative et les intervalles de confiance qui y sont associés. La formule d'estimation qu'on recommande est cells qui est donnée par la méthode de la probabilité maximale en supposant que la variance (de l'erreur) qui reste dans les observations soit égale sur les deux navires. On applique les formules à des données expérimentales recueillies à des profondeurs variées. Les limites de confiance sur b vont se rapprochant avec l'accroissement de la profondeur.

INTRODUCTION

When two ships are engaged concurrently on an acoustic survey, the performance of their equipment may be compared by means of an inter-ship calibration (Rottingen, 1978). This is done by the ships steaming in close formation while a series of paired echo integrator readings is obtained. Suppose that n such pairs are obtained and that x_i and y_i, i = 1, 2, ..., n, are the observed values for ship A and B respectively. It is assumed that x_i and y_i are echo integrator readings corresponding to some conceptual density of fish D say. However, because of characteristics specific to each ship's sonar equipment and because of the way the fish are spatially distributed, the observed values x_i and y_i will not be equal. It may be realistic to assume that, in the absence of measurement and sampling errors, both x_i and y_i are related in some stochastic sense to the true density D. Under specific assumptions, to be discussed below, the observations x_i and y_i can be regarded as giving rise to points which deviate from an "underlying" relationship

$$Y = \alpha + \beta X \tag{1}$$

the parameters α and β being unknown. Relationships of this type are known as functional or structural relationships depending on the statistical properties attributed to the observations x_i and y_i.

The objective of an inter-ship calibration exercise is to estimate the relationship (1), that is, to estimate the parameters α and β along with their standard errors or appropriate confidence limits. The latter are of importance in that they provide a means of indicating whether different survey results from the two ships might be explained by differences in performance rather than real differences in fish density.

THEORY

One model which may be postulated is that in which x_i and y_i are random variates with true values X_i and Y_i; that is

$$x_i = X_i + d_i \qquad y_i = Y_i + e_i \tag{2}$$

where the true (expected) values X_i and Y_i are functionally related according to the law

$$Y = \alpha + \beta X \tag{3}$$

This model would hold if it were accepted that the expected values of x_i and y_i are both linearly related to the true density D. That is

$$X = u_1 + v_1 D \qquad Y = u_2 + v_2 D \tag{4}$$

so that

$$(X - u_1)/v_1 = D = (Y - u_2)/v_2$$

which implies that

$$Y = \alpha + \beta X$$

The difficulty in estimating the parameters α and β of equation (3) arises from the fact that they refer to a relationship between quantities which cannot be measured without error. This problem has been studied extensively by many authors, particularly Lindley (1947). A variety of estimates of the slope parameter β has been proposed corresponding to different assumptions about the statistical properties of the errors d and e. Some of these models have been discussed by Pope and Shanks (1982). In the present context it will be assumed that d and e are uncorrelated with each other and that their variances, var(d) and var(e), are constant. It will also be assumed that neither d nor e are (serially) correlated in time. That is

$$\text{var}(d_i) = \sigma_d^2$$
$$\text{var}(e_i) = \sigma_e^2 \qquad (5)$$
$$\text{cov}(d_i\ e_j) = o$$

for all values of i and j.

If $\sigma_d^2 = o$ (implying that $x = X$) then β is appropriately estimated by the slope of the regression line of y on x, namely

$$b = S(xy)/S(x^2)$$

while if $\sigma_e^2 = o$ (implying $y = Y$), then the slope of the regression line of x on y, namely

$$b = S(xy)/S(y^2)$$

should be used. If the errors are assumed to be normally distributed, these estimators are maximum likelihood estimators, otherwise they are least squares estimators. In either case they are best linear unbiased estimators. However, when neither σ_d^2 nor σ_e^2 are identically zero these estimators are no longer appropriate. Three different estimators for this model are considered and compared in this paper. As the constant term α (the offset) is, in each case estimated by

$$a = \bar{y} - b\bar{x} \qquad (6)$$

whatever slope estimator is employed, only different estimators of β will be considered in what follows.

Use of the method of maximum likelihood to give an estimator of the parameter β has been studied by Lindley (1947) who noted that the method fails to give a solution unless additional information about the variances σ_d^2 and σ_e^2 is available. In particular, if the ratio $\sigma_e^2 / \sigma_d^2 = \lambda(\text{say})$ is known, the estimator of β is given as the solution of the quadratic equation

$$S(xy)\beta^2 - [S(y^2) - \lambda S(x^2)]\beta - \lambda\ S(xy) = 0 \qquad (7)$$

The reason for the failure of the method of maximum likelihood has been shown by Solari (1969) to be due to the fact that the solutions of the maximum likelihood equations correspond to a saddle-point of the likelihood surface, not to a maximum.

In the absence of any information to the contrary, a value of $\lambda = 1$ would seem a reasonable assumption in intercalibration work. The estimate of the slope is then, by solving (7),

$$b = [\ S(y^2) - S(x^2) + \{[S(y^2) - S(x^2)]^2 + 4\ S^2(xy)\}^{1/2}]/2\ S(xy) \qquad (8)$$

The construction of a confidence interval for this estimator of the slope has been studied by Creasy (1956). She worked not with the slope and its estimator but with the corresponding angles between the line and the positive direction of the X-axis, ie with $\theta = \text{arc tan}\ \beta$ and $\hat{\theta} = \text{arc tan}\ b$. An <u>approximate</u> confidence interval for θ is given by

$$\hat{\theta} \pm \tfrac{1}{2} \arcsin \{4t^2 [S(x^2)S(y^2) - S^2(xy)] / [(n-2)[S(x^2) - S(y^2)]^2 + 4S^2(xy)]\}^{\frac{1}{2}} \quad (9)$$

In the above expression t is the appropriate value of Student's t for (n-2) degree of freedom. The confidence interval for b is given by the tangents of the angle limits from (9).

An entirely different estimator has been proposed by Wald (1940). He considered an estimator of the form

$$b_w = (\bar{y}_2 - \bar{y}_1)/(\bar{x}_2 - \bar{x}_1) \quad (10)$$

where $(\bar{x}_1, \bar{y}_1)$ is the 'centre of gravity' of half the observations and $(\bar{x}_2, \bar{y}_2)$ the centre of gravity of the other half. Wald showed that if the (x, y) values included in the first half correspond to those whose X-values are all smaller than the X-values associated with the points in the other half, then b_w is a consistent estimator of β. (That is b tends in probability to β as $n \to \infty$). Such a partitioning of the data can be achieved provided the x-values are separated by distances greater than their errors so that the observed x's are in the same order as the true X's. Unfortunately, the error variance in inter-ship calibration data is likely to be large so that incorrect allocation of some of the observations to their correct group may be common. There is no _a priori_ reason to group the observations according to the order of data from one ship rather than the other. Two slope estimates are obtained by grouping the observations on the order of x and y values in turn. The validity of Wald's method may be gauged by comparing these slope estimates. A significant difference would suggest that too many observations had been allocated to the wrong group.

Wald also derived a method for calculating a confidence interval for the slope. These limits are given by the two roots of the quadratic equation

$$\beta^2 \{g\, s(x^2) - (\bar{x}_2 - \bar{x}_1)^2\} + 2\beta \{b_w (\bar{x}_2 - \bar{x}_1)^2 - g\, s(xy)\} + \{g\, s(y^2) - b_w^2 (\bar{x}_2 - \bar{x}_1)^2\} = 0 \quad (11)$$

where

$$g = 4\, t^2/n$$

$$s(x^2) = \{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2\} / (n-2)$$

$$s(y^2) = \{\Sigma(y_1 - \bar{y}_1)^2 + \Sigma(y_2 - \bar{y}_2)^2\} / (n-2) \quad (12)$$

$$s(xy) = \{\Sigma (x_1 - \bar{x}_1)(y_1 - \bar{y}_1) + \Sigma(x_2 - \bar{x}_2)(y_2 - \bar{y}_2)\} / (n-2)$$

The use of three or more groups, instead of just two, has been proposed by other authors but it is not clear whether such procedures are always more efficient.

A third estimator of the slope which has been advocated is the so-called geometric mean regression slope (Ricker, 1973). This estimator is the square-root of the ratio of the slope of the regression of y on x to that of the regression of x on y; that is

$$b_{gm} = \pm \{S(y^2)/S(x^2)\}^{\frac{1}{2}} \quad (13)$$

where the sign is chosen to make $(b.S(xy)) > 0$. This estimator has been criticised by a number of authors (eg Jolicoeur, 1975; Sprent and Dolby, 1980; Pope and Shanks, 1982). It can be shown that, if in (7) the value of λ is put equal to $S(y^2)/S(x^2)$, then the estimator given by applying the method of maximum likelihood is equal to b_{gm}. The data by themselves cannot be used to say whether the assumption that $\sigma_e^2 / \sigma_d^2 = S(y^2)/S(x^2)$ is a reasonable one or not. If it is then b_{gm} does correspond to a maximum. Intuitively the geometric mean regression slope is attractive since it corresponds to a line bisecting the angle between the two ordinary regression lines. When this angle is small the geometric mean regression line is likely to be close to the true relationship sought, but this is, of course, also true for the regression lines themselves and indeed for an eye-fitted line.

By considering the effect of small uncorrelated variations in the observed parameters, and assuming that λ is estimated by $S(y^2)/S(x^2)$, we have derived the following approximate confidence interval for b_{gm}.

$$b_{gm} \pm t\, b_{gm} \{2[1-S(xy)/ \{S(x^2)\, S(y^2)\}^{½}] /n\}^{½} \qquad (14)$$

CALCULATIONS WITH EXPERIMENTAL DATA

The research vessels "Magnus Heinason" (Faroe Islands) and "Scotia" (UK) conducted an inter-ship calibration during a blue whiting survey in April 1982. The three analysis methods discussed above have been compared using the data from this calibration.

The echo integrators were set to record the energy returned from three depth channels: (1) 0 to 240m, (2) 240m to the sea bed and (3)from the sea bed to 50m above. The ships covered a 40 mile track aling the edge of the continental shelf north of Shetland. The water depth varied from 340m to 500m. Thus the first channel did not overlap with the others, while the second channel included all the echoes from the third. The integrators were read at one mile intervals. For each depth channel, therefore, 40 paired measurements were obtained. The data from the two ships were converted to the same units, tonnes per square km, using the same reference target strength of -34dB re 1kg.

The three sets of data are plotted in Figures 1 to 3. Also shown are the fitted lines according to the geometric mean regression (GMR) and the maximum likelihood (ML) for $\lambda = 1$ methods. The various slope estimates and the associated confidence limits are shown in Table I.

DISCUSSION

It is evident from the trend of the confidence limits that there is less random scatter in the data from deeper water. This is to be expected. The conical shape of the acoustic beam means that the beams will overlap more as the depth increases, so that a larger proportion of the targets will be detected by both ships. The effect is well known, but it has provided sets of data with convenient differences in residual variance for the purposes of this paper.

Visual inspection of the data suggests that the residual variances are not constant, contrary to one assumption that is common to all the methods described here. The concentration of points near zero shows relatively little variability, particularly in channel 3. This is not surprising since the data are constrained to be positive and many of the points are near zero, corresponding to little or no fish.

Wald's estimator does not compare well with the others. The confidence limits are wider and the slope estimates are different when the groups are selected on y values instead of x values, especially in channel 1.

The slopes estimates are consistently and significantly less than 1. There was therefore a real difference in performance between the two vessels. The poor weather conditions at the time of the calibration may be partly responsible. Wind and ship motion induce air bubbles in the water which attenuate the acoustic beam (Dalen and Lovik, 1981). "Magnus Heinason" had a hull mounted transducer, while the "Scotia" transducer was in a towed body which would reduce the aeration problem to some extent.

The slope estimates tend to increase with the depth. This effect could indicate a difference in the time varied gain of the echosounders.

The ML slope estimate is consistently lower than the GMR, although the difference is slight except in channel 1. The channel 1 data are too variable to be of much use for calibration, and outliers may have had a disproportionate effect.

CONCLUSIONS

Wald's grouping method is inadequate for the analysis of inter-ship calibration data. It is little better than simple linear regression.

Both the ML and GMR methods suggest a significant difference in performance between the two ships, a slightly larger difference being indicated by the ML estimate. The GMR method has the intuitively unsatisfactory feature that the slope estimate takes no account of the association between x and y. Consider the example of the same quantity being observed by two measurement systems which introduce random uncorrelated errors. The expected value of S(xy) is then zero. The ML method has the satisfactory feature that the result is indeterminate when s(xy) is zero.

We recommend therefore that maximum likelihood theory with $\lambda = 1$ should be adopted as the preferred method for the analysis of inter-ship calibration data. The slope and the associated confidence limits are estimated using equations (8-9).

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TABLE I

Comparison of slope estimates from the "Scotia" (x) and "Magnus Heinason" (y) data. L and H are the 95% confidence limits (t = 2.02, degrees of freedom = 38)

Method		Channel 1	Channel 2	Channel 3
Maximum likelihood ($\lambda = 1$)	b H L	0.25 0.40 0.17	0.51 0.72 0.33	0.56 0.71 0.43
Geometric mean regression	b H L	0.40 0.53 0.28	0.57 0.67 0.46	0.58 0.65 0.51
Wald - group of x values	b H L	0.30 0.54 0.11	0.38 0.54 0.17	0.55 0.67 0.43
Wald - group of y values	b H L	0.75 10.92 0.40	0.62 0.99 0.42	0.59 0.74 0.48

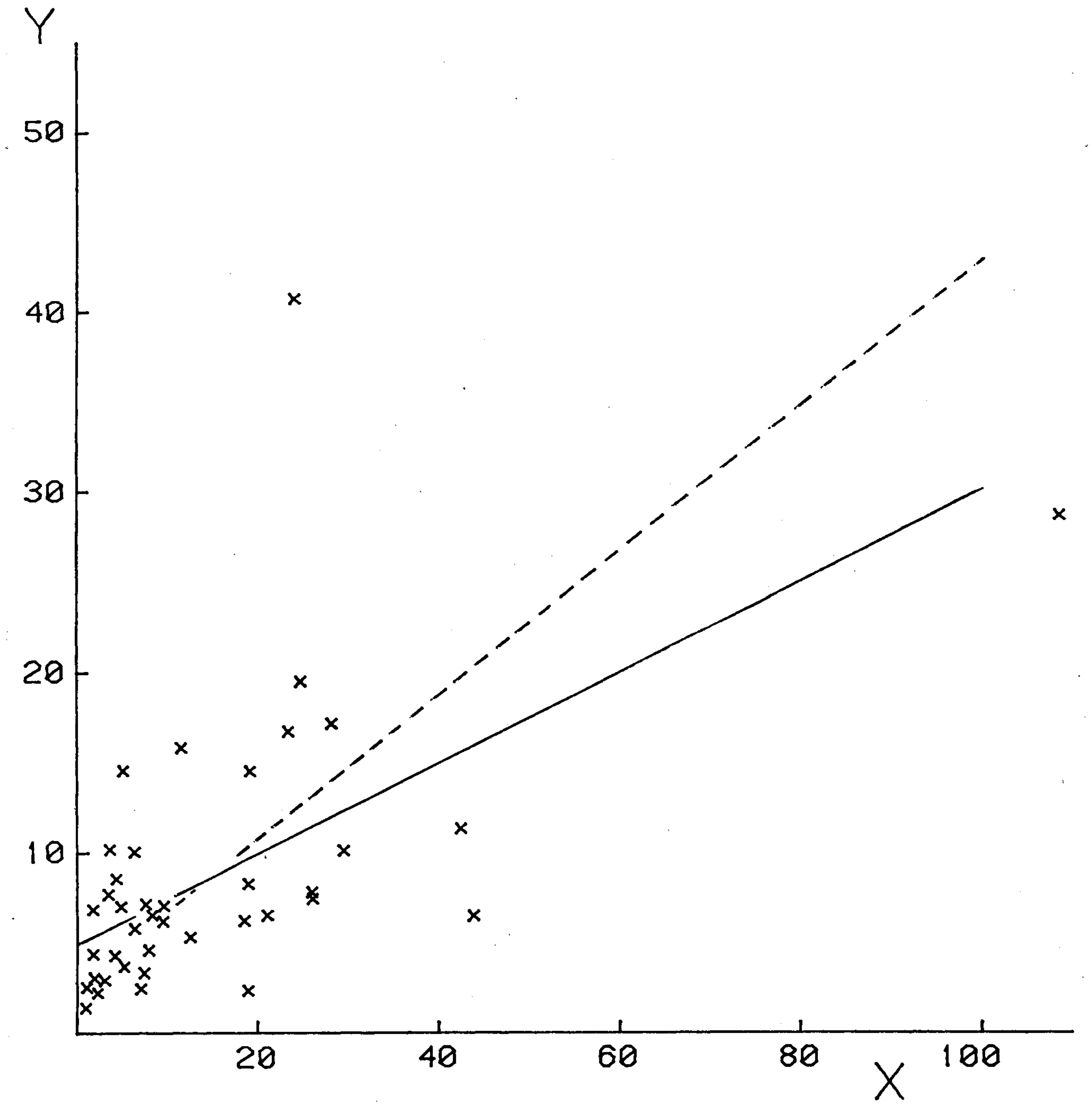

FIGURE 1. CHANNEL 1 DATA. X = 'SCOTIA' AND Y = 'MAGNUS HEINASON'. THE BEST FITS ACCORDING TO THE GEOMETRIC MEAN REGRESSION AND MAXIMUM LIKELIHOOD METHODS ARE THE DASHED AND SOLID LINES RESPECTIVELY.

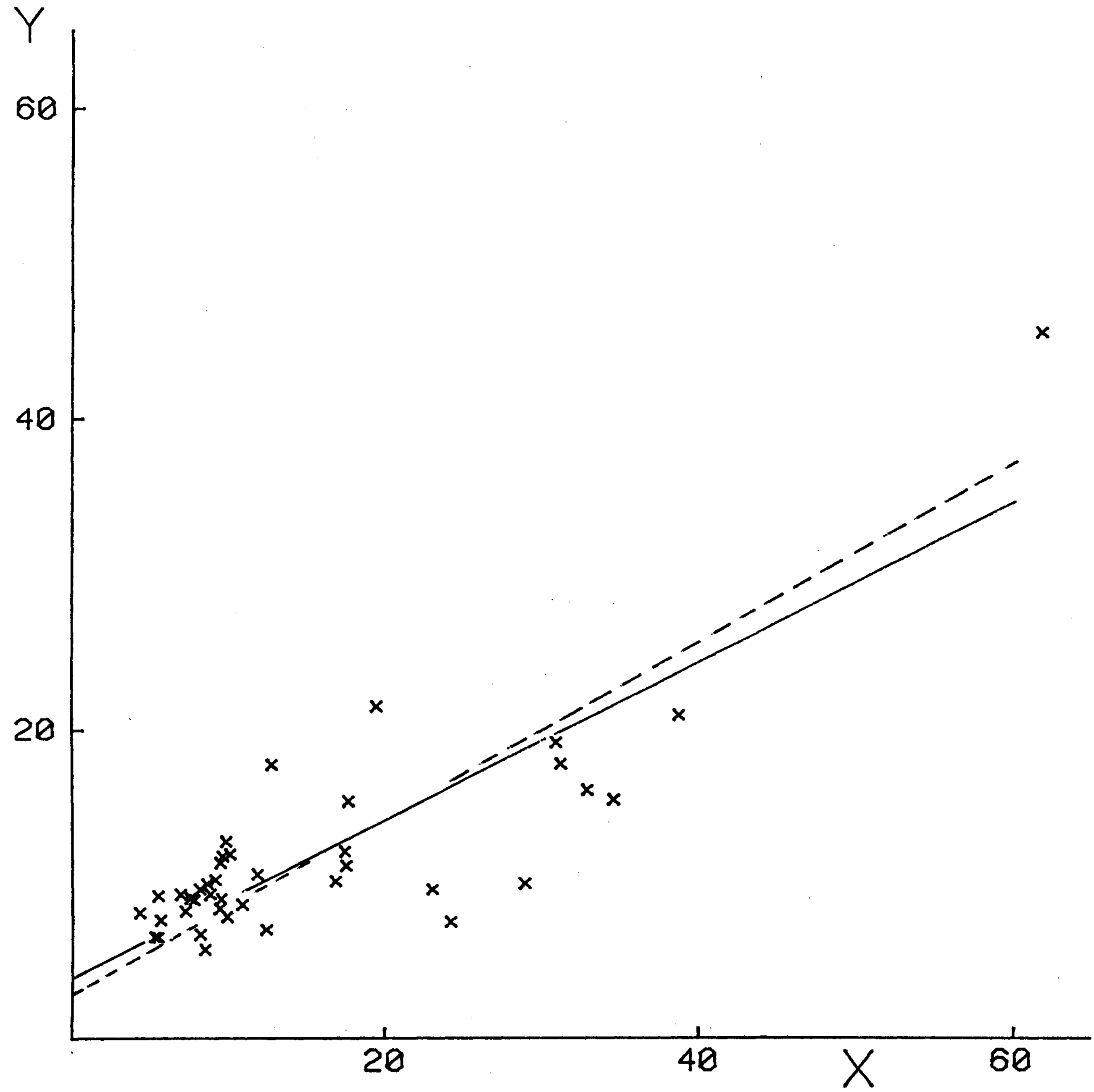

FIGURE 2. CHANNEL 2 DATA. X = 'SCOTIA' AND Y = 'MAGNUS HEINASON'. THE BEST FITS ACCORDING TO THE GEOMETRIC MEAN REGRESSION AND MAXIMUM LIKELIHOOD METHODS ARE THE DASHED AND SOLID LINES RESPECTIVELY.

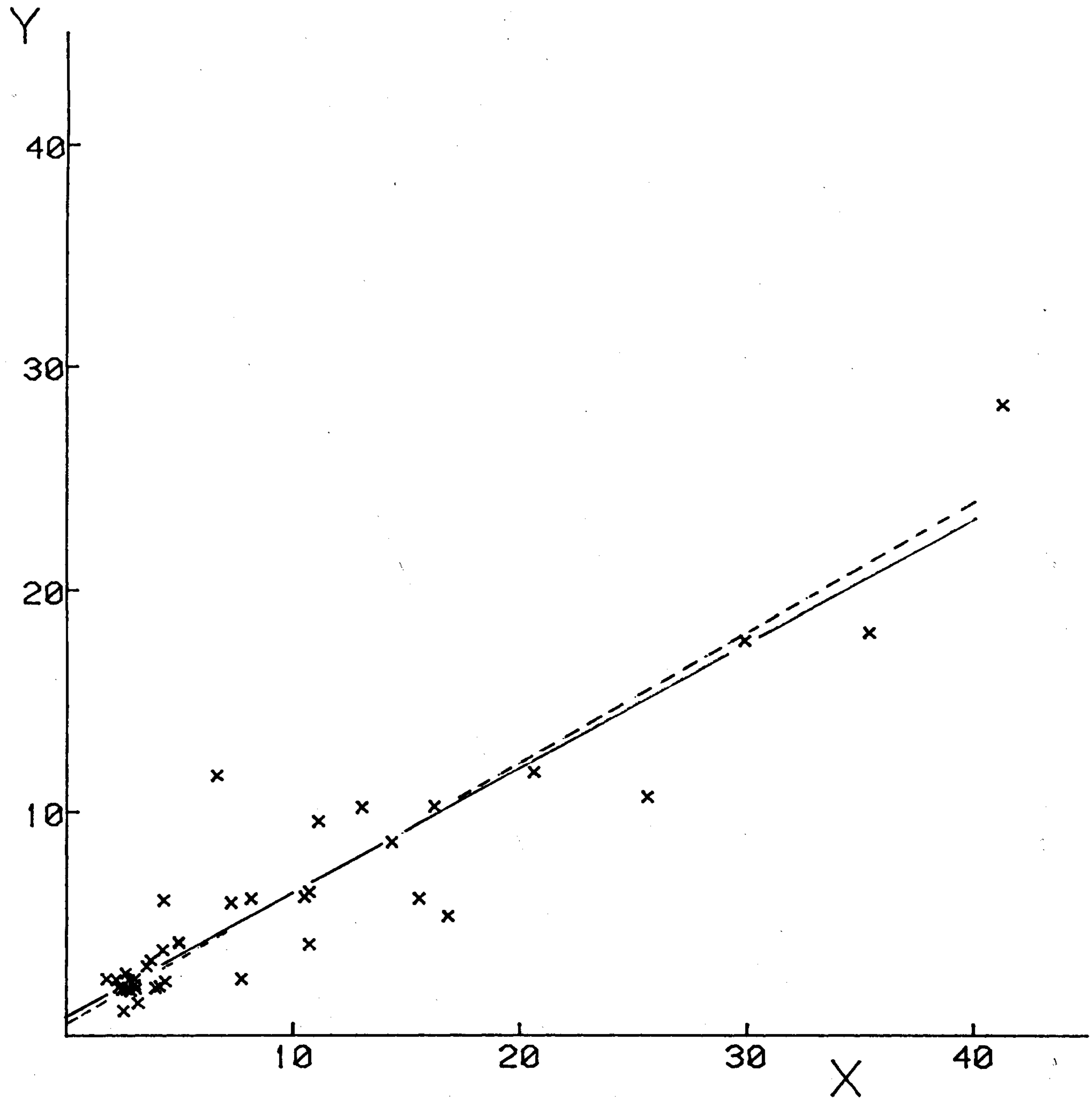

FIGURE 3. CHANNEL 3 DATA. X = 'SCOTIA' AND Y = 'MAGNUS HEINASON'. THE BEST FITS ACCORDING TO THE GEOMETRIC MEAN REGRESSION AND MAXIMUM LIKELIHOOD METHODS ARE THE DASHED AND SOLID LINES RESPECTIVELY.