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Estimation of strength of yearclasses and of fishing efficiency based on environmental assumptions in a self-generating population model.

by
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Abstract

A birth-death model with birth being dependent on the environment is set up . The estimation procedure is discussed and an example the North Sea Herring is briefly treated for illustration purpose.

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Contents

Notation

$c_{a\,j}^{t\,m}$	Catch in numbers at the time interval [t; t+1] with fishing gear m in the area j, the age of the fish being a.
$n_{a\,l}^{t}$	Number of fish at time t of age a belonging to stock l.
$\alpha_{j\,l}^{t\,a}$	Distribution factor e.g. the fraction of the year t where fish of age a belonging to stock l is in area j.
$I_{t\,j}^{m}$	Fishing intensity with gear m at time t; t+1 at area j.
Z	Total mortality coefficient.
M	Natural death mortality coefficient.
$\bar{T}$	Mean temperature of the last 6 months of the year.
E	Spawning potential No. of eggs
S_a	Fecundity of age a.

Introduction

The model which shall be described in this paper is an attempt to give a simple way of doing forecasts consistent with data for a period covering a yearspan before the time from where the forecast shall begin.

For this task a simple birth-death model is set up with two sources of death e.g. fishing- and natural death. The birth part is taken to be a function of the stocksize, age composition of the stock and of the environment. The environment parameters can be very difficult to forecast, but seems necessary for fitting the model to a reasonable description of the data available.

The parameters should be estimated by setting up the least square expression for the data available. The data may differ between applications and therefore no conclusive can be said about estimation.

As an example calculations using data for the herring of the North Sea is shown. It is merely an illustration and not a final analysis which is presented.

Formulation of model

1. The catch $c^{t\,m}_{a\,j}$ at time t with fishing gear m in area j of fish with age a is written

$$C^{t\,m}_{a\,j} = \sum_{l \in \text{stock}} n^{t}_{a\,l} \frac{\varphi^{m}_{a} I^{m}_{t\,j}}{Z^{t}_{a\,j}} \left(1 - e^{-\alpha^{t\,a}_{j\,l} Z^{t}_{a\,j}}\right)$$

Where $n^{t}_{a\,l}$ is the stock l in numbers of age a at time t, $\alpha^{t\,a}_{j\,l}$ is the distribution of stock l in area j, at time t of age a, φ^{m}_{a} is the factor applied to the fishing intensity $I^{m}_{t\,j}$ where m denotes the fishing gear and t and j as above. $Z^{t}_{a\,j}$ is the total mortality coefficient

$$Z^{t}_{a\,j} = M_a + \sum \varphi^{m}_{a} I^{m}_{t\,j}$$

M_a is the natural mortality only dependent of age.
The relation between $n^{t}_{a\,l}$ and $n^{t+1}_{a+1,l}$ is taken

$$n^{t}_{a\,l} = \sum_{j \in \text{area}} n^{t+1}_{a+1,l}\, e^{-\alpha^{t\,a}_{j\,l} Z^{t}_{a\,j}}$$

in agreement with the formula for the catch $c^{t\,m}_{a\,j}$

The critical point of the model is to make it self-generating by the introduction of a recruitment, Beverton and Holt (1957) gives

$$n_{ol}^{t+1} = \frac{\alpha E_l^t}{E_l^t + \beta}$$

Where E_1^t is the spawning potential e.g. number of eggs at time t of stock 1. The fecundity is given as a function, spawning potential per fish depending of the age S_a and we have

$$E_l^t = \sum_{a \in age} S_a \cdot n_{al}^t$$

The above formulation only applies to constant environment and is thus not able to account for the very large observed scatter in the stocksize of the yearclasses.

We therefore state

$$n_{ol}^{t+1} = \frac{\alpha E_l^t}{E_l^t + \beta} f(\text{environment})$$

and shall for every fish we try to use the model on specify the function f.

This point is the essiential one, if we are not able to get a good description of the stock - recruitment - relationship the entire model can not be applied. However if a model of the type discussed here can be made working, one is far better off when making assessment and the uncertainties in the estimates of catches can be much more detailed studied.

The number of parameters introduced is high and it should be advisable before using the model to introduce a number of assumptions regarding the actual fish in question, which will reduce the number of parameters.

Estimation of the Parameters

The unknown parameters of the model is the following

M_a

φ_a^m

$n_{a\,1}^{o}$

and

α, β, f(environment)

These parameters and function shall be estimated.
In principle the least square expression is set up for the catches

$$\sum_{\text{Observations}} \left[C_{a\,J}^{t\,m}(\text{observed}) - C_{a\,J}^{t\,m}(\text{calculated}) \right]^2$$

This is a complicated task and worse data present may put restrictions on how many parameters can be estimated. Other sources of data f.ex. measurements of stock-size from research vessels may be available and should be taken into account.
Also the set of parameters one is specially interested in calls for an individual treatment. Therefore no conclusive can be said on estimation,- it will be necessary to put up the least square expression in each case and then apply a standard tecknique for finding the minimum.

One method which has been succesfully used in a number of cases is the linearition of the equation-system and then solve the problem by iterative corrections to the set of unknowns.

Whatever method for solving the least square problem is chosen, one will nearly always be dealing with rather complicated expressions or very lengthy calculations or both, which will call for a computer. It should be advisable to start thinking in computers instead of spending time at the desk calculator first.

In the next paragraph a very simple exsample is shown.

North Sea Herring. An Illustration.

In this paragraph the model shall be illustrated using data on the North Sea Herring. Unless otherwise stated data has been extracted from Report of the North Sea Herring Assessment Working Group, this meeting.

First we will go over all the parameters

1) Natural Mortality M_a

The age dependency can be introduced by

$$M_a = \frac{\gamma_M}{W_a^{2/3}}$$

Since little is known of M_a it is often taken constant throughout the lifetime. Anyhow the above formula is adopted. With W_∞=271 and M = .0859 for old herrings one find

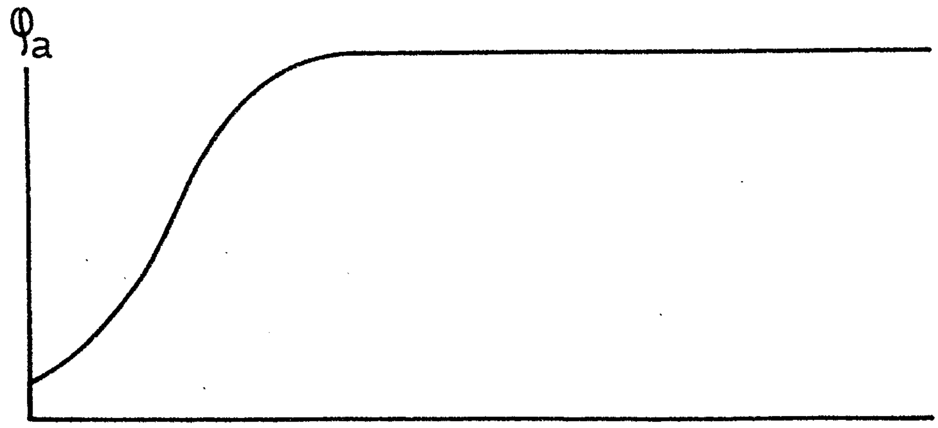
$$\gamma_M = 3.6$$

W_a is calculated using the Berthalauffy equation.

2) Fishing Mortality Coefficient φ_a^m

The problem of different fishing gears is treated, only one gear will be taken into account.

The age dependency φ_a is taken as a cumulated normal distribution



the formula being

$$\varphi_a = \frac{\Psi}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{L_a} e^{-\frac{(\ell-\bar{\ell})^2}{2\sigma^2}}\, d\ell$$

σ^2 being the variance and $\bar{I}$ the mean value of the normal distribution. L_a is the length at age a taken from the Berthalauffy equation. The ψ is given in units of reciproc fishing intensity.

3) Fishing Intensity $I^m_{t\ j}$

The North Sea is treated as one area and shall then just give I_t in arbitrary units.

	Year				
	1963	1964	1965	1966	1967
Fishing Intensity	.17	.31	.76	.72	.85

This determine $\psi = 1$

4) Stock - Recruitment Relation

On fig.1 the logaritm of the estimated number of recruits (O-ringers) is plotted against $1/\bar{T}$, $\bar{T}$ being the mean temperature of the period july-december both included, at the position 53° 45'N 01° 54'E. (Annales Biologiques). The years 1954 to 1960 is shown.

This suggests

$$f(\text{environment}) \sim e^{\gamma/\bar{T}}$$

with

$$\alpha = 8e^{-145.3}$$

$$\beta = 0$$

$$\gamma = 4.4_{10^4}$$

5) Berthalanffy Parameters W_∞, k, to

The Berthalanffy equation

$$W_a = W_\infty (1 - e^{-k(a-t_0)})^3$$

have been used with following parameters

$$W = 271$$
$$k = .377$$
$$t_o = 1.53$$

6) Age Composition at Beginning of 1963.
Calculated using Cohort Analysis

	Winterrings									
	0	1	2	3	4	5	6	7	8	9
Stock size 10^9	8.9	6.6	12.3	.8	.1	.1	1.4	.2	.1	.1

7) Result of calculation.

With the set of parameters given in the sections above the expected catches in numbers were calculated. In the table below the calculated ones are compared with the observed.

	Numbers 10^9 Catches calculated	observed
1963	3.7	4.2
1964	6.6	6.0
1965	12.4	8.3
1966	8.4	6.1
1967	8.0	5.4

It should be noted that the year classes 1963 to-66 is at average or below. $\bar{T} = 12\,^{\circ}C$, $\alpha_{Je}^{ta} = 1$

Conclusion

A birth-death model was set up and the recruitment is a function of the environment. The estimation should be at least square procedure. The illustration using data on the North Sea herring gives an order of magnitude of the expected accuracy which can be ashieved given that the effort is known. It is about 30%.

Reference

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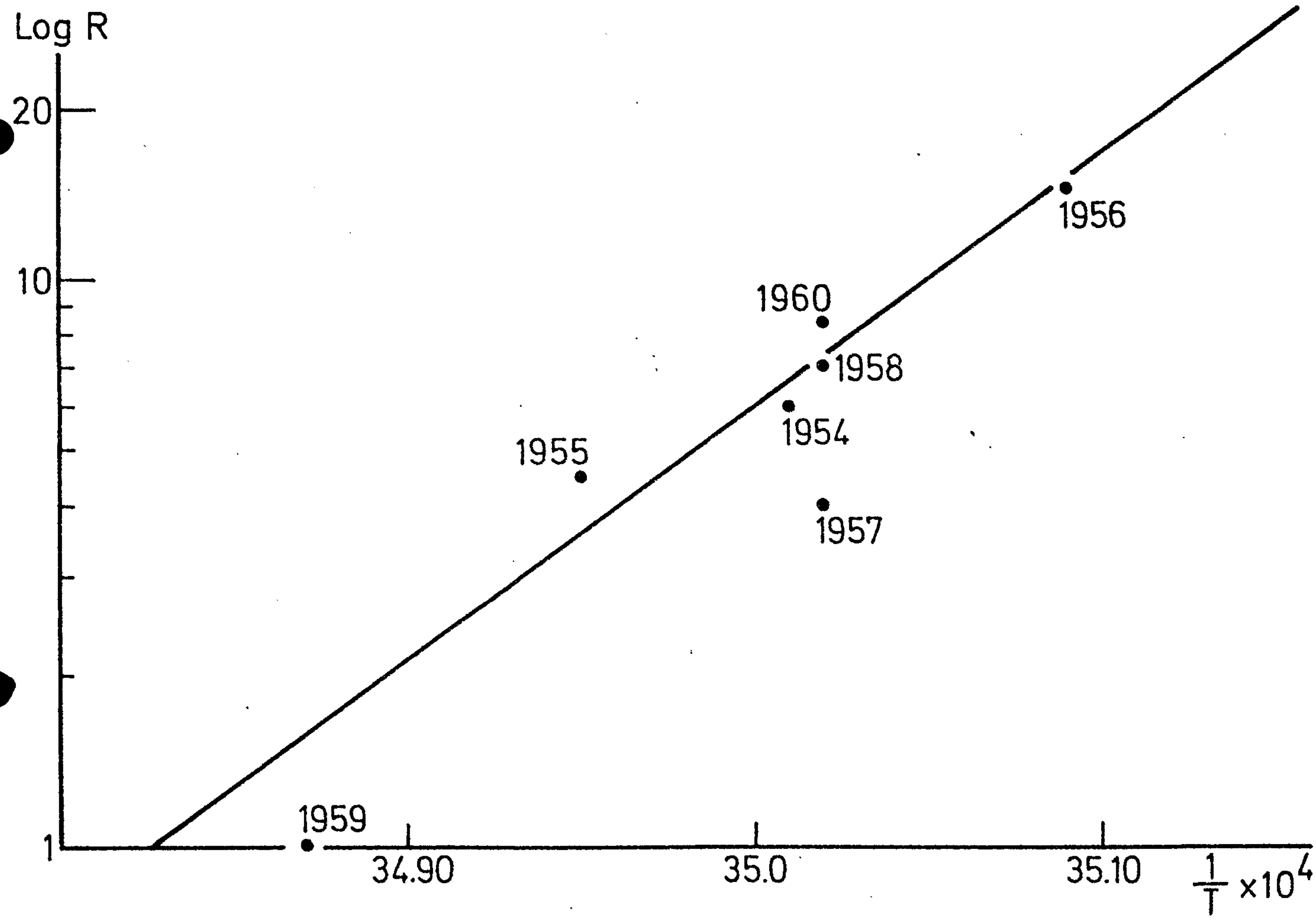

Figure 1. The logaritm of the number of recruits (0 winterrings) in billions plotted against the reciprocal of the mean temperature of the period july-december at position 53° 45' N, 01° 54' E. The years 1954 - 1960 is shown.