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UTILIZATION OF THE ICELANDIC COD STOCK IN A MULTISPECIES CONTEXT

by

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ABSTRACT

Harvesting of cod, capelin and shrimp in the Icelandic marine ecosystem is analysed with respect to the probable effects of different harvesting strategies on yield and biomass of each species. Simulations are used to investigate the probability profiles of these quantities. taking into account inaccuracies in assessments and uncertainties in predictions. Potential yield using different harvesting strategies is also investigated using deterministic models.

1. Introduction

In 1992, the Minister of Fisheries asked the Marine Research Institute to "prepare proposals for how individual fish stocks should be exploited with the aim of achieving the maximum yield over the long term from Icelandic waters". Subsequently, the Marine Research Institute (MRI) asked the National Economic Institute (NEI) to cooperate on this task. For this purpose a joint working group of the MRI and the NEI was established in January of 1993.[1] Preliminary results of that analysis, taking into account single-stock dynamics and economic considerations, were given in Baldursson *et al*, 1993.

This paper describes a continuation of work done under the auspices of the working group, where several multispecies concerns have been addressed.

The biological side is modelled with simple extensions of the cohort model described in Beverton and Holt (1957) with a recruitment function of the Ricker form, modified to account for potential cannibalism as in Pope and Woolner (1981) and Pope (1992). Growth of cod is modelled using equations linking growth to the size of the capelin stock as in Steinarsson and Stefánsson (1991). Predation by cod on capelin affects not only cod growth but also capelin catches and this is modelled through a simple scaling of natural mortality on juvenile capelin, along the lines of Pope and Woolner (1981). Predation of shrimp by cod is modelled using a modified version of the biomass model in Stefánsson *et al*, 1993, which is a stock-production model incorporating measured recruitment.

The model is made stochastic by adding random components to account for recruitment variability and other sources of uncertainty using the suggestions set out in Anon. (1993d).

It should be pointed out that the Icelandic cod stock has been severely depleted and is currently far below benchmarks such as the maximum sustainable yield level. Therefore, it is not only necessary to study what the desirable long term management policies are, but one must also try to solve the dynamic problem of how to rehabilitate the stock.

A sensitivity analysis of the effects of some departures from the assumptions of the model is conducted. This includes some major deviations from the usual assumptions of fish stock assessments, such as a test of what happens if the certain factors in the ecosystem cycle through good and bad periods, resulting e.g. in periods of poor and good recruitment of cod and/or capelin.

In Sections 2 and 3 the single species and multispecies biological models, respectively, will be described. In Section 4 the various measures of benefits used are discussed. Section 5 presents the results of the stochastic simulations. The sensitivity analysis is conducted in Section 6 and, finally, some conclusions are drawn in Sections 7 and 8.

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[1] Apart from the authors of this paper the members of the working group are:
Mr. Brynjolfur Bjarnason, Chairman of the board of MRI, who is Cairman of the group
Mr. Jakob Jakobsson, Managing Director of the MRI
Mr. Thordur Fridjonsson, Managing Director of the NEI

2. Notation

Notation will clearly become somewhat complex when several stocks are considered. Classical notation will be used as far as possible, for such items as total or exploitable biomass (*B*), spawning stock biomass (*S*) and recruitment (*R*). When needed the year in question will be denoted with a subscript *y* so that R_y is the recruitment in year *y*.

Superscripts are used to indicate species with cod denoted by G (*Gadus morhua*), capelin by M (*Mallotus villosus*) and shrimp by P (*Pandalus borealis*). Thus cod recruitment in year *y* is denoted by R_y^G. In cases when the notation applies only to one of the stocks the superscript may be dropped. The following table illustrates some of the notation used, with further definitions given in the text as needed.

Symbol	Explanation	Symbol	Explanation	Symbol	Explanation
B_y^G	Exploitable biomass of cod (*Gadus morhua*) in year *y*.	B_y^M	Biomass of capelin (*Mallotus villosus*) 1. January of year *y*.	B_y^P	Shrimp (*Pandalus borealis*) biomass
		$B_{y,0}^M$	Biomass of capelin 1. July of year *y*.		
p_{ay}^G	Proportion of cod mature at age *a* in year *y*.				
R_y^G	Cod recruitment in numbers in year *y*.	R_y^M	Recruitment of capelin as the number of 1 year olds on 1. July in year *y* of the year before the fishery on the yearclass starts.	R_y^P	Shrimp recruitment in year *y* (index).
S_y^G	Spawning stock biomass of cod in year *y*.	S_y^M	Capelin SSB, i.e. the target SSB, nominally 400 thousand tonnes.		

$C^G_{a,y}$	Cod catch in numbers of age a in year y.				
Y^G_y	Cod landings in year y.	$Y^M_{y,1}$	Capelin landings, 1. January to the end of the season (March).	Y^P_y	Shrimp landings in year y.
		$Y^M_{y,0}$	Capelin landings, 1.July-31.Desember of year y.		
J^G_y	Index of immature (2+) component of the cod stock in year y.				
w^G_{ay} v^G_{ay} u^G_{ay}	Mean weight values of cod age a in year y.				
F^G_y	Overall fishing mortality on cod in year y.	M^M_y F^M_y	Natural mortality of capelin yearclasses in year y. Proportion of available capelin caught in fall in year y.		

Greek letters will throughout denote parameters which need to be estimated.

3. Single species models

The three species discussed in this paper are interrelated through predation by cod, which affects the growth of cod and the natural mortality of the other species.

3.1 The Model of the Cod Stock

In the cohort model described by Beverton and Holt (1993) the fish stock is assumed to consist of a number of age groups. Here it is assumed that fish will enter the stock at age 3 and will at most reach the age of 14. Thus, in this case the model encompasses 12 age groups. The number of a year old fish at the beginning of year y is denoted by $N_{a,y}$. For fish age 3 and older, the number of fish at the beginning of the next year is given by:

$$N_{a+1,y+1} = N_{a,y} \exp(-s_{a,y} F^G_y - M_{a,y}) \qquad 3 \le a \le 13$$

$$N_{15,y} = 0$$

where F_y^G is fishing mortality in year y, $s_{a,y}$ is the fishing pattern by age group and $M_{a,y}$ is the natural mortality in each age group, which here is assumed to be equal to 0.2 for all age groups every year. Thus, $N_{a,y} - N_{a+1,y+1}$ fish have disappeared from the age group during the year y. They are divided among fish caught and dead from natural causes in relation to the mortality parameters so that the number of fish in the age group caught during year y according to the Baranov equation (Baranov, 1918) is:

$$C_{a,y}^G = s_{a,y}F_y^G / (s_{a,y}F_y^G + M_{a,y}) \cdot (1 - \exp(-s_{a,y}F_y^G - M_{a,y}))N_{a,y}$$

and the catch in weight units during year y is:

$$Y_y^G = \sum_{a=3}^{14} w_{a,y}^G C_{a,y}^G$$

where $w_{a,y}^G$ is the average weight of fish aged a in year y in the catch (see multispecies section). The exploitable biomass is given by:

$$B_y^G = \sum_{a=4}^{14} v_{a,y}^G s_{a,y} N_{a,y}$$

where $v_{a,y}^G$ is the weight of fish of age a in year y in the sea. Here it has been assumed that $v_{a,y}^G = w_{a,y}^G$. The spawning stock during the spawning period is:

$$S_y^G = \sum_{a=3}^{14} u_{a,y}^G p_{a,y} N_{a,y} \exp(-s_{a,y}F_y^G f_a - M_{a,y} m_a)$$

where $u_{a,y}^G$ is the weight of fish of age a in year y during the spawning period, $p_{a,y}$ is the fraction of fish of reproductive capacity of age a in year y, f_a is the fraction of the total annual fishing mortality inflicted on age group a before spawning commences and m_a is the fraction of the natural mortality inflicted on age group a before spawning.

As recommended in Anon. (1993d), a stock-recruitment relationship is introduced in order to incorporate the potential effect of a severaly reduced spawning stock. The form chosen is based on the Ricker functional form, $R(S) = \alpha S e^{-S/k}$, where α and k are positive constants. This base model was augmented by accounting for potential cannibalism (or competition) resulting in:

$$R_y^G = \varsigma S_y^G e^{-S_y^G / K} e^{-\xi J_{y+1}^G}$$

This model is based on Pope and Woolner (1981) and has further been used e.g. by Bogstad *et al* (1993), and Pope (1992). In this equation J denotes the juvenile (immature) biomass defined by

$$J_y = \sum_{a=3}^{14} u_{a,y}^G (1-p_{a,y}) N_{a,y} \exp(-s_{a,y} F_y^G f_a - M_{a,y} m_a).$$

Thus, in year y, J_y^G denotes the biomass of immature fish of age 3 or older. It is more appropriate to assume that cod of age 2+ may be able to consume the 0-group cod, i.e. to use a 2-year lag between the cannibal and its prey. Thus, J_{y+1}^G is used as an index of the biomass of immature fish of age 2+ in the year, y, that the yearclass is spawned. Naturally, there are some caveats to this definition, but it has been chosen for conformity with the definition of the spawning stock, i.e. the index is taken at time of spawning so the sum of J and S will give the total available stock at that time. Although it is somewhat contrary to common belief, the uncorrected time shift in the index computation assumes that M on the 2-group is similar to natural mortality on older fish. There is no concrete evidence to the contrary, however, and hence this approach is taken.

For 1993-1995 projected recruitment is based on available data on stock sizes (Anon., 1993a and c) along with information on catches in 1993 and 1994 survey indices:

Table 3.1 Recruitment 1993-1995

Millions of individuals

Year	Recruitment
1993	144
1994	73
1995	130

The initial values of parameters are presented in Table 2.2. Maturity and weight at age and the selection pattern are the estimated values for 1993. The initial stock size in 1994 by age group is based on Anon. (1993a), revised in accordance with available information from surveys and catches since that time.

Table 3.2 Initial values of parameters for the cod stock

	M	p	w	u	s	f	m	N94
3	0.2	0.08	1.30	1.04	0.18	0.09	0.25	73.00
4	0.2	0.25	1.78	1.57	0.35	0.18	0.25	99.09
5	0.2	0.47	2.50	2.52	0.62	0.25	0.25	79.07
6	0.2	0.71	3.32	3.61	0.90	0.30	0.25	18.49
7	0.2	0.94	4.49	4.87	1.07	0.38	0.25	9.73
8	0.2	0.98	5.69	6.15	1.13	0.44	0.25	1.99
9	0.2	0.97	6.95	7.54	1.13	0.48	0.25	1.45
10	0.2	0.97	8.21	8.84	1.13	0.48	0.25	1.42
11	0.2	1.00	12.75	11.09	1.13	0.48	0.25	0.00
12	0.2	1.00	13.45	12.00	1.13	0.48	0.25	0.00
13	0.2	1.00	15.88	14.40	1.13	0.48	0.25	0.00
14	0.2	1.00	11.76	18.38	1.13	0.48	0.25	0.00

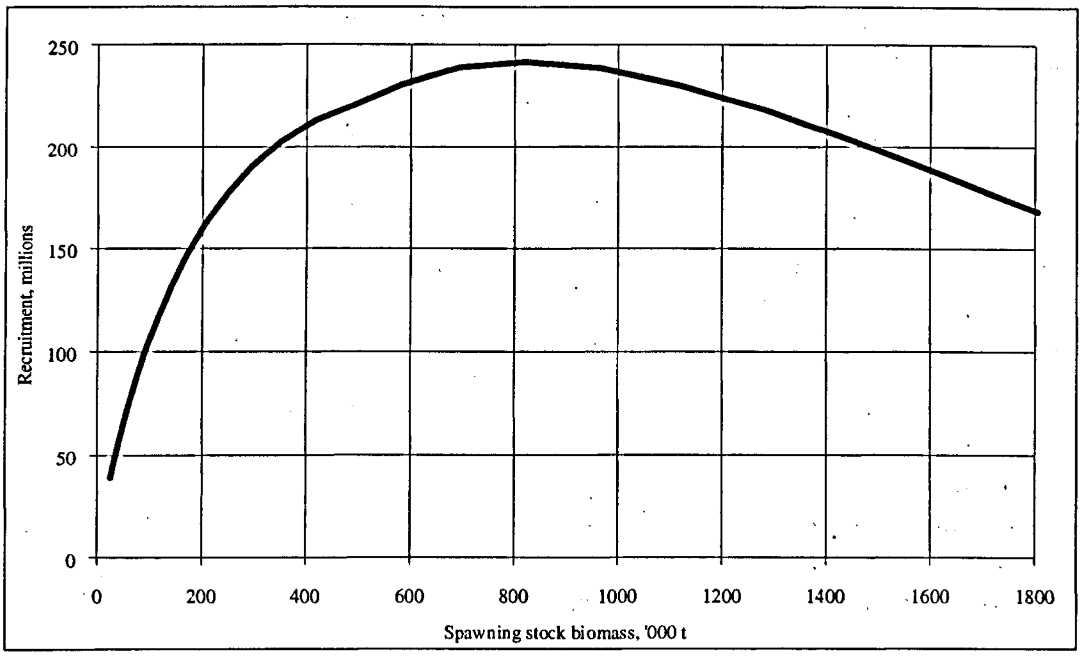
The maturity parameters were kept constant between years from 1995. Values in 1994 were interpolations of the long-term values and the initial values (c.f. Table 2.3). The values of parameters in the recruitment function were results from maximum likelihood estimation based on assuming logR to follow a Gaussian density:

$$\varsigma = 1.46$$
$$K = 764$$
$$\xi = 0.69$$

The function gives a prediction of recruitment in terms of millions of individuals. There is considerable uncertainty in the parameter estimates and this is incorporated in the stochastic simulations by using the estimated variance-covariance function from the above estimation procedure. Thus, at the start of each simulation a set of stock-recruitment parameters was generated from assuming a multivariate normal distribution of $(\varsigma, 1/K, \xi)$. This set is then used to generate the time series of recruitment in that particular simulation.

The above formula explicitly incorporates the juvenile biomass of cod as well as the spawning stock biomass. The stock can be projected forward assuming a constant fishing mortality, resulting in a constant (or average) recruitment, spawning stock biomass and juvenile abundance. Thus a functional relationship is obtained between the equilibrium state of any two of the three quantities. This approach can be used to depict the recruitment as a function of the spawning stock biomass.

Fig 3.1. Spawning stock and recruitment relationship for cod.



In the stochastic simulations several of the above quantities are taken to be random variables. An attempt was made to take account of measurement error in the stock size and variability in future recruitment as well as weight at age.

The initial stock size is based on random perturbations from the estimates given in Anon. (1994). The stock-estimation procedure used e.g. in Anon. (1993c) is based on estimating a single fishing mortality multiplier but fixing the selection pattern to equal the average of some earlier years. Hence the stock estimates for the different age groups will be highly positively correlated in practice. The level of correlation will vary depending on the assessment method used, but is likely to be high regardless of the particulars of the method, simply due to possible year effects in the survey data.The random perturbations are therefore obtained by drawing from a log-normal distribution with a standard deviation of 0.15 and multiplying the entire initital stock-numbers at age by this random number, except for the youngest yearclass.

To generate random fluctuations in recruitment the recruitment function was multiplied by a log-normal random variable with a 35% standard deviation, which is in accordance with the observed CV of recruitment. The recruitment estimates for 1993-1995 were multiplied by a log-normally distributed random variable with a 25% standard deviation.

The above needs to be modified since there is more information (from groundfish surveys) on age groups 3-5 in the initial year than that contained only in the selection pattern or the stock-recruitment curve. This information is incorporated by assuming the 3 year old cod in numbers to be estimated with a 20% standard error based on the groundfish survey and further assuming that the VPA-generated fishing mortality table is correct. Thus the number of 3-5 year old fish in the beginning of 1993 is projected forward from the number as 3-group.

In section 4.1 methods or procedures for setting TACs for the cod stock are considered. These rules must in practice be based on estimated quantities such as spawning stock biomass (SSB) or other measures of the state of the stock. Account was taken of the inevitable error in the spawning stock estimate by assuming that the estimated SSB was unbiased, but subject to a log-normal error with a 15% standard deviation. In other words, in year y, the SSB was multiplied by a lognormally distributed random number before applying a catch control law.

The selection pattern was assumed to be known and fixed, equal to the selection pattern estimated for 1993. This same selection pattern was used for generating populations and catches.

This choice of simulating the estimation and recruitment variation is one of several possibilities mentioned in Anon (1993d). The particular choice of the standard errors is along the lines indicated in Guðmundsson (1987) and the recruitment variation indicated above.

Initial values for maturity and weight at age of cod are given in Table 2.3 (Anon. 1993a). As seen e.g. in Steinarsson and Stefánsson (1991), growth can be quite variable for this stock and part of this variability is due to multispecies interactions. It is assumed that the mean weights at age will develop in an orderly fashion as described in the multispecies section.

3.2 A model for the capelin stock

The model for the capelin stock is driven only by catches and natural mortalities as described in the following equations which are further described below.

$$B_{y-1,0}^{M} = R_{y-1}^{M} e^{-12M_{y-1}^{M}}$$

$$B_{y}^{M} = B_{y-1,0}^{M} - C_{y-1,0}^{M}$$

$$S_{y}^{M} = B_{y}^{M} - C_{y,1}^{M}$$

The stock size and natural mortality have been measured using acoustic surveys (Vilhjálmsson 1994). Natural mortality is further assumed to change according to the consumption of capelin by cod, as described in the multispecies section.

The capelin is mainly harvested during the months July-March, leading to a natural split into two seasons, July-December and January-March. During each of the seasons reduction capacity may be a limiting factor. Also, the price of the meal from each season will vary.

Capelin recruitment needs some careful consideration. It should be noted that the capelin stock declined to very low level twice in the 1980s, reaching minimum levels in 1982 and 1989. Since there are some indications that the same may have occurred in the 1970s, one approach to modelling recruitment to this stock is by assuming that recruitment cycles in some fashion, reaching very low levels every few years. The approach taken is therefore to assume sinusoid cyclic recruitment with a period of 7 years. For stochastic simulations the period is taken to follow a distribution which is uniform on 5, 6, 7, 8 and 9 years and lognormal random noise with s.e.=0.20 is added to the recruitment.

3.3 A model for the shrimp stock

The dynamic stock biomass model of Stefánsson *et al* (1993) is used, albeit with some minor changes. Thus it is assumed that the biomass of shrimp in year *y+1* will be the same as in the current year, after adding recruitment and subtracting catches and consumption by cod. Recruitment and consumption are estimated based on indices of the two quantities.

$$B_{y+1}^{P} = B_{y}^{P} + \eta R_{y}^{P} - Y_{y}^{P} - \lambda J_{y}^{G}.$$

The equation contains two explicit unknown parameters along with the virgin biomass. Knowledge of these three parameters would allow the computation of the entire biomass series. Assuming further that CPUE for shrimp is proportional to average biomass permits the estimation of the parameters involved using least squares.

	CPUE (*pandalus*)	R (*pandalus*)	Y (*pandalus*)	Juveniles (*gadus*)	B (*pandalus*)
					101.91
1978	65.8	2461	1.4	861	102.97
1979	75.7	2313	1.1	872	103.44
1980	79.8	4747	3.1	880	114.33
1981	77.6	3212	2.1	704	120.42
1982	76.4	1909	1.7	623	121.15
1983	85.0	4368	6.1	584	130.60
1984	86.0	2418	12.2	604	123.68
1985	93.0	3930	12.2	573	124.90
1986	89.0	4943	17.1	770	124.11
1987	77.5	4309	24.6	936	110.58
1988	65.8	4089	20.7	840	100.96
1989	68.0	4994	18.1	606	101.37
1990	82.8	8180	19.4	402	119.29
1991	87.4	8406	26.1	412	131.52
1992	84.3	6376	27.4	263	133.79
1993	95.5	7192	30.5	338	136.29

3.4 Multi-Species and Density Effects

Models used previously in assessments of the Icelandic cod stock have been mainly single species models (Anon. 1993, Baldursson *et al,* 1993) with only the growth of cod affected by the size of the capelin stock in the short term.

If the recent increase in shrimp catches is due to the poor condition of the cod stock which in part feeds on shrimp, one would expect the recovery of the cod stock to lead to a contraction in shrimp fishing. In a multi-species model with interaction between the shrimp stock on the one hand and the cod stock on the other the increase in the cod stock would entail costs associated with the decline in shrimp fishing. Against the reduced costs in cod fishing as a result of a larger stock one would have to balance the increased costs in shrimp fishing as a result of the reduced shrimp stock.

As noted in Baldursson *et al* (1993), care needs to be taken when models indicate that the SSB of cod may increase to e.g. over 1 million tonnes, which will happen if low fishing mortalities are assumed. In particular, density dependent weight at age may well appear as may several other factors. In order to accomodate this, density dependence along the lines indicated in Schopka (1993) is implemented. Using those results, it is possible that no effect of stock on weight at age is seen when the SSB is at 200 thousand tonnes whereas the weight at age drops to 70% of the maximum when the SSB reaches a million tonnes.

As indicated in Steinarsson and Stefánsson (1986), mean weight at age for cod can be predicted with

$$\tilde{w}_{a+1,y+1} = (\alpha + \beta w_{ay} + \gamma B_y^M).$$

This can then be modified by accounting for density dependence with:

$$w_{a+1,y+1} = p_y^G \tilde{w}_{a+1,y+1},$$

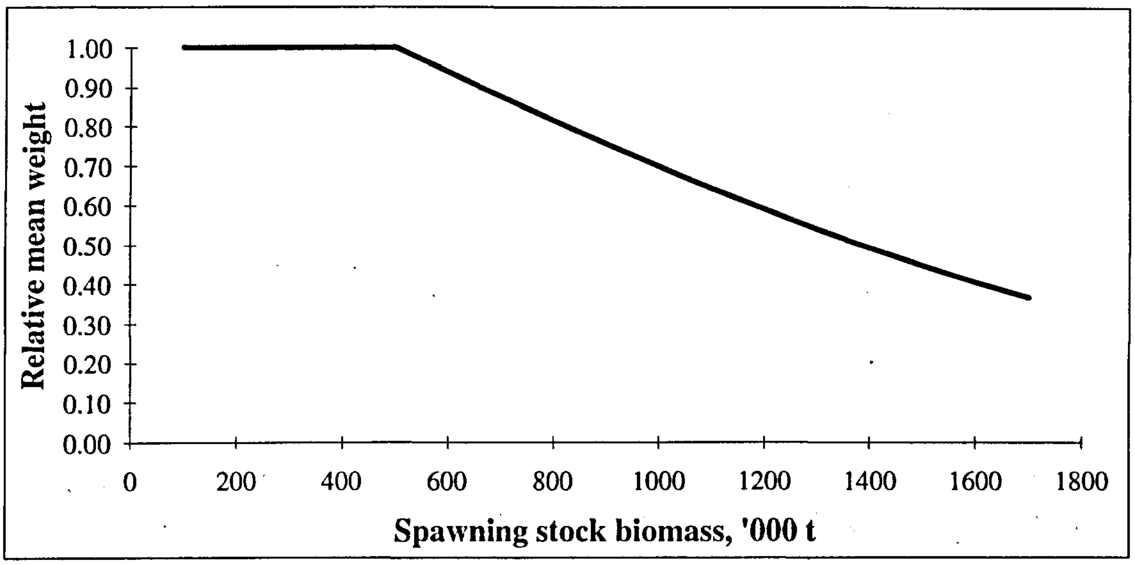
where the fraction

$$p_y^G = \begin{cases} 1 & S < S^{G,\min} \\ \dfrac{2}{1+e^{\phi(S_y^G - S^{G,\min})/(S^{G,\max} - S^{G,\min})}} & S^{G,\min} < S < S^{G,\max} \\ 0 & S > S^{G,\max} \end{cases}$$

is a coefficient which describes the effect of density dependence. By setting the coefficients in the equation to to ϕ =11.8, $S^{G,\max}$=10000 and $S^{G,\min}$=500, this will result in a proportion which is about 1 at 500 thousand tonnes and about 0.7 at 1 million tonnes. The extreme setting with ϕ =0 results in density-independent individual growth.

The following figure illustrates how the density dependent effects affect the weights at age, reducing them from 100% at low stock levels.

Fig. 3.2. Reduction of weight at age as a function of SSB.



It is clear that this approach will limit the computed growth potential in the cod stock. Although it is clear that such a limit exists, it is equally clear that assumptions about weight reduction at high stock sizes will be hard to justify fully since past data on the relationship between stock size and weight at age is confounded with environmental effects.

It should be noted that the effect of the capelin stock on the growth of the cod can only be seen for ages 4-8. For other ages a different model is needed and average growth is applied to obtain a predicted weigth at age before applying the density dependent effect as above.

One consequence of this approach is that if the cod stock increases a lot, the size of the capelin stock will exert little effect on the cod stock, since the density dependent effects will dominate.

The above approach is used both for mean weights at age in the SSB and the mean weight at age in the catches.

For **capelin**, natural mortality is assumed to vary in some accordance with the size of the cod stock. The average natural mortality for capelin in the years 1978-1987 was estimated using acoustic measurements to be $M_0^M = 0.035$ (Vilhjálmsson, 1994). It is quite natural to assume that natural mortality increase from this value as the cod stock increases in size since predation of capelin is to a large extent by cod (Pálsson, 1983). Based on these concerns the natural mortality of capelin is assumed to fluctuate according to:

$$M_y^M = M_0^M (S_y^G / \overline{S}^G)$$

Thus the natural mortality of capelin is scaled according to the size of the cod stock. Although a *predator stock* could be defined for this purpose, the SSB of cod is used. In the above, $\bar{S}^G$ denotes the average spawning stock biomass of cod during the years 1978-1987.

For **shrimp**, recruitment has been shown to be lower on average when the immature part of the cod stock increases. In particular, Stefánsson *et al* (1993) provide a linear relationship between J_y^G and R_y^P, and such a relationship is used in the forward projections of the *Pandalus* stock in this paper.

4 Catch control laws and utility

In order to evaluate different methods of harvesting the stocks involved, formal procedures need to be defined for the take from each stock. Such a procedure or method will be called a catch control law, a quota rule or a management procedure.

4.1 Cod

A good management procedure needs to meet a number of conditions. First, it is desirable that the rule leads to a long term equilibrium for the stock which is desirable from biological and economic considerations, including a high economic yield. Second, the rule should even out fluctuations. Third, it should be possible to explain and justify the rule to managers and the general public. Finally, the rule should minimize the probability of a collapse in the stock.

If a management procedure can be explained to managers, then that should also be considered beneficial. Rules based on fishing mortalities are in this respect considerably inferior to rules directly relating catches to quantities such as spawning stock biomass or exploitable biomass. A final aspect relates to the comparison of the management procedure to a method which is already in place. For a large number of stocks there is current no formal management objective. However, there is usually some form of management and in some cases the current method of managing the stock can be put into a parametric framework. In such instances it may be feasible to formulate alternative strategies in terms of parametric deviations from the current setting. For example, a common claim is that "catches must not drop below x t". In this situation it is interesting to compare various management procedures which explicitly try to attain a certain minimum catch, even when the stock size is very low.

Based on this notion, the approach has been taken to formulate a catch control law which has the basic principle to increase catches as the stock increases. In order to avoid excess catches simply due to measurement errors in the stock estimation, an upper limit is placed on the catches. A lower limit (zero or greater) is also set in order to permit the parametric testing of the effect of attempts to maintain minimal catches in spite of low stock sizes.

The form of the management procedure used here is the following:

$$Q_y = \min(\delta Q_{y-1} + (1-\delta)\max(Q_{\min}, \min(Q_{\max}, a(\hat{S}_{y-1} - b))), Q_{1.5,y})$$

where $Q_{1.5,y}$ is the catch obtained by setting the true F in year y to 1.5.

Thus, the procedure can be described as a result of the application of a sequence of several steps. A fixed fraction, a, of the estimated spawning stock in excess of a certain limit, b, is calculated. In these calculations a=45% and b=50 thousand tons. This provides the basic catch limit, which needs to be modified according to 4 criteria. The catch is lowered to certain limit ("roof") if a higher value than this upper limit was obtained in the first step, but raised to a "floor", if a lower value than this lower limit was obtained in the first step. The roof is $Q_{max} = 450$ thousand tons and several values were tested for the floor, Q_{min} . Severe changes from one year to the next are quite plausible simply due to the high level of variability in the stock estimates, and hence the δ coefficient can set to a value between 0 and 1 in order to reduce interannual variability. The value 0.5 is used in the trials presented.

Finally, the condition that the fishing mortality rate does not exceed 1.5 was imposed. The value of 1.5 is a somewhat arbitrary choice, but this value is used to reflect an estimate of the maximum fishing mortality which the fleet can excert on the stock. Three minimum tonnage figures were tested: 125, 175 and 225 thousand tons.

In order to examine the properties of the procedure, some form of steady state analyses are useful. Since the capelin stock cycles, the concept of steady state is somewhat vague, but the average quantities for the last seven years of the deterministic projections will be used.

Figure 4.1 Steady state catch and the management procedure.

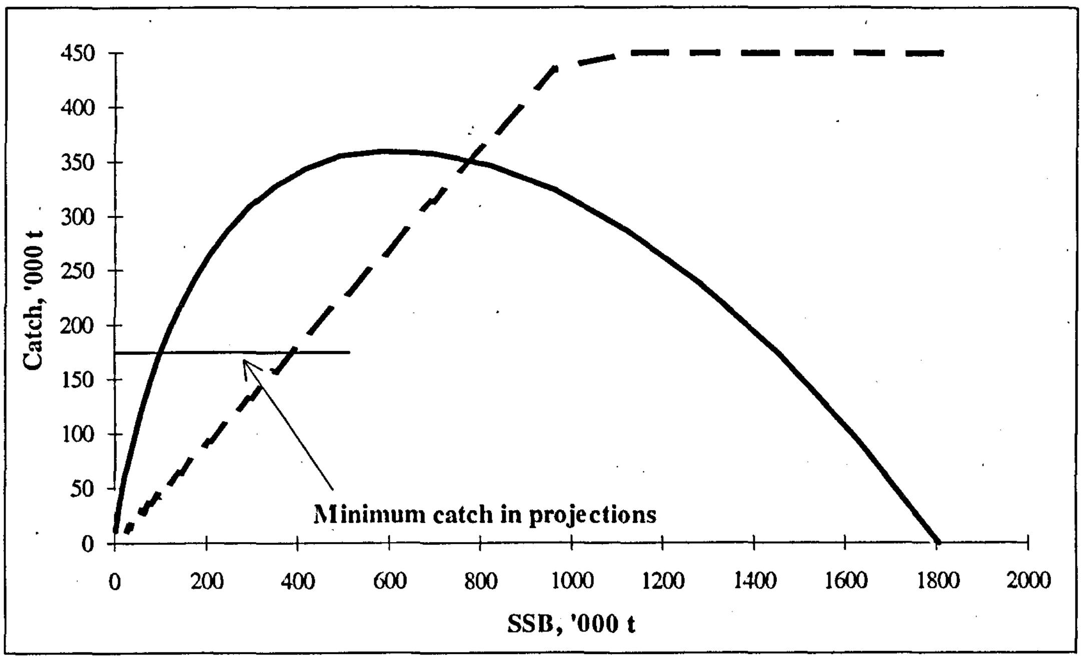
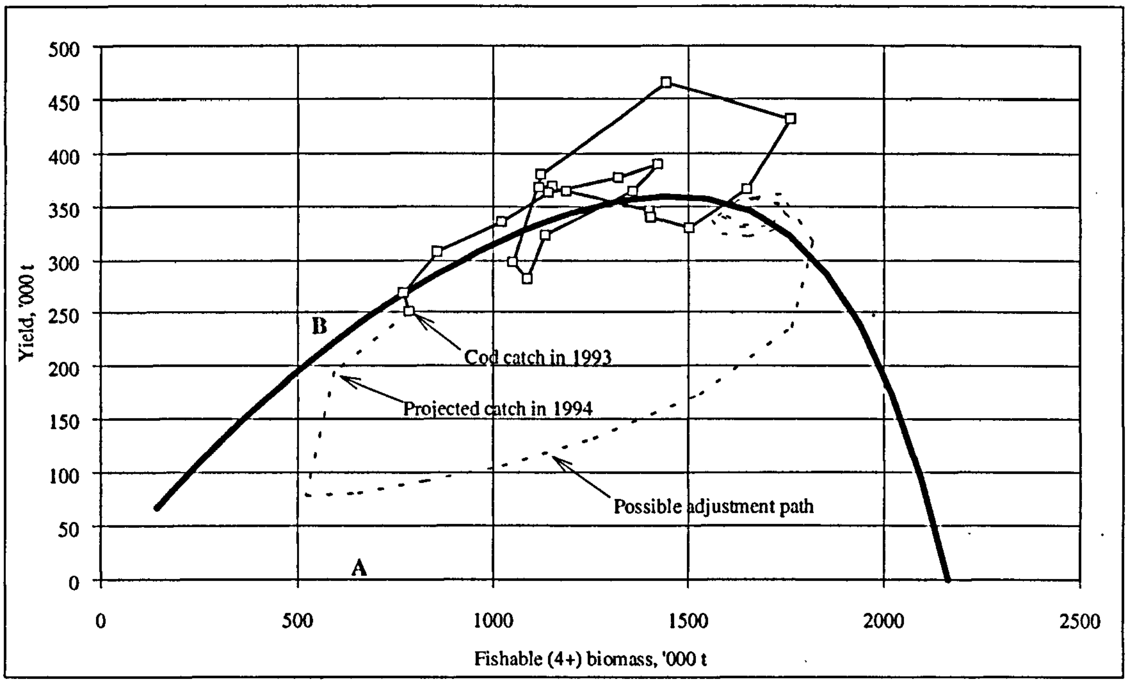


Figure 4.1 shows the management procedure as a function of the estimated spawning stock, assuming a minimum catch of 175 thousand tons. Limitations on changes between years are not taken into account in Figure 4.1, nor is the curve which becomes binding at low stock sizes when the catch control law is limited by $F=1.5$.

It is interesting to examine the historical development of the Icelandic cod stock in the context of the steady state catch. This is done is Figure 4.2 with fishable biomass as a measure of stock size. The figure illustrates clearly how the stock has declined along with catches more or less consistently above the steady state catch curve. The clarity of this picture is somewhat surprising in view of the vast simplification embodied in the steady state catch curve. Also shown is the deterministically projected adjustment path using the catch procedure of Figure 4.1 with zero minimum catch.

Figure 4.2 Steady state catch and historical catch against SSB



4.2 Capelin

Utilisation of the capelin stock has been implemented in the light of international agreements . The capelin stock is harvested with a strategy which aims for a target spawning stock biomass of 400 thousand tonnes.

The fishing year for capelin starts roughly in July of one year and ends in March of the next. In the present model the growth of cod depends on the abundance of capelin on January 1. Further, the spawning biomass of 400 thousand tonnes is assumed to be enough to avoid recruitment failure due to a possible stock-recruitment relationship. Thus there is no interaction between the amount of allowed fishing in the January-March period and any other component of the model. Assuming that capelin fishing is profitable as a single-fleet, single-species fishery, it follows that any rule concerning the fishery in January-March will be optimized by catching the entire TAC which remains for that period.

Thus the capelin fishery as modelled here only contains a single unknown, the proportion of the TAC to be taken in fall, F_y^M. The profitability of the fall fishery will depend on the relationship between the price of capelin in fall and spring, the relationship between the price of capelin in fall and the potential worth of the same capelin to the growth of cod, *etc.*

During the past decade or so, natural mortality and individual growth of capelin have roughly cancelled during the September-March period. It is partly for this reason that predation is only taken into the natural mortality up to the time when the capelin enters the fishery. Alternative models should be considered as a continuation of the present work. Such models might include heavier predation effects during the season and continuous effects of capelin biomass on cod growth, rather than the point-impact at January 1. implemented here.

4.3 Shrimp

Currently the offshore shrimp stock is harvested using a procedure which aims to keep the stock at a long-term average level. In order to simplify this, the approach taken was to use a catch-control law which keeps the biomass constant from one year to the next, if possible. In the present biological model there is no incorporation of density-dependent factors which may lead to increased production at one level of the shrimp biomass rather than at another. Thus there is no long-term gain in the model of changing the stock size and hence the implemented strategy will be adequate within the current model.

4.4 Profit and utility functions

The three species considered in this paper fetch different prices in the market and the costs of catching and processing are different. To consider overall effects of different harvesting strategies it is necessary to take account of these differences in prices and costs. The prices used here are 110 Icelandic kronors (IKR) per kg for cod, 140 IKR/kg for shrimp and the price of capelin is taken to be 7.75 IKR/kg during the period January-March and 9.90 IKR/kg during July-December. There is assumed some price elasticity for cod, but other product prices are assumed constant. The CPUE is assumed to increase with bigger exploitable biomass in the case of cod and shrimp. The elasticity of CPUE with respect to eploitable biomass is assumed to be 0.7 for cod and 1.0 for shrimp. The CPUE in the capelin fisheries are assumed independent of the size of the stock, but different during the two periods mentioned. The elasticity of the CPUE for cod is important for the results of the calculations, but as the catch rule for shrimp is such that the exploitable biomass remains almost constant the elasticity for shrimp does not matter so much. The most recent estimates of unit cost in fishing and processing were used and these costs were assumed constant during the simulation period.

The decision of the optimal harvesting strategy is done in two stages. Stage one is to choose the long run optimum stock size and catch levels. The criteria which is used is the maximization of the sum of discounted profits. The second stage is to decide on an adjustment period and to choose the optimal harvesting strategy during this adjustment period on the basis of criteria which take some account of the present situation of unemployment and excess capacity. The criteria take also some account of the fact that it is not possible to avoid completely some oscillations in consumption because of the oscillations in income which follows from the chosen harvesting strategy. This was done by using income instead of consumption in the utility funciton. (Further details in Baldursson et al., 1993.) Aversion to fluctuations in consumption is taken into account by using a concave utility function. The form assumed for the instantaneous utility of consumption is the constant relative risk (or fluctuation) aversion form:

$$u(c) = (c^{1-\sigma} - 1)/(1-\sigma)$$

where c is consumption and $\sigma = -u''(c)\cdot c/u'(c)$ is the coefficient of relative risk aversion. The function is scaled so that it changes to log(c) when $\sigma \to 1$. In the calculation values of σ ranging from 0 to 6 were used. Common values in econometric studies are in the vicinity of 2 (Lucas, 1987). When consumption (c) has been replaced by income and σ is 0 (zero aversion to fluctuations in consumption), the maximization of the sum of discounted utility is equivalent to the maximization of the sum of discounted income. In this particular model this is equivalent to maximizing discounted net export earnings or discounted domestic value added.

5 Steady State Characteristics and basic conclusions

5.1 Biological results

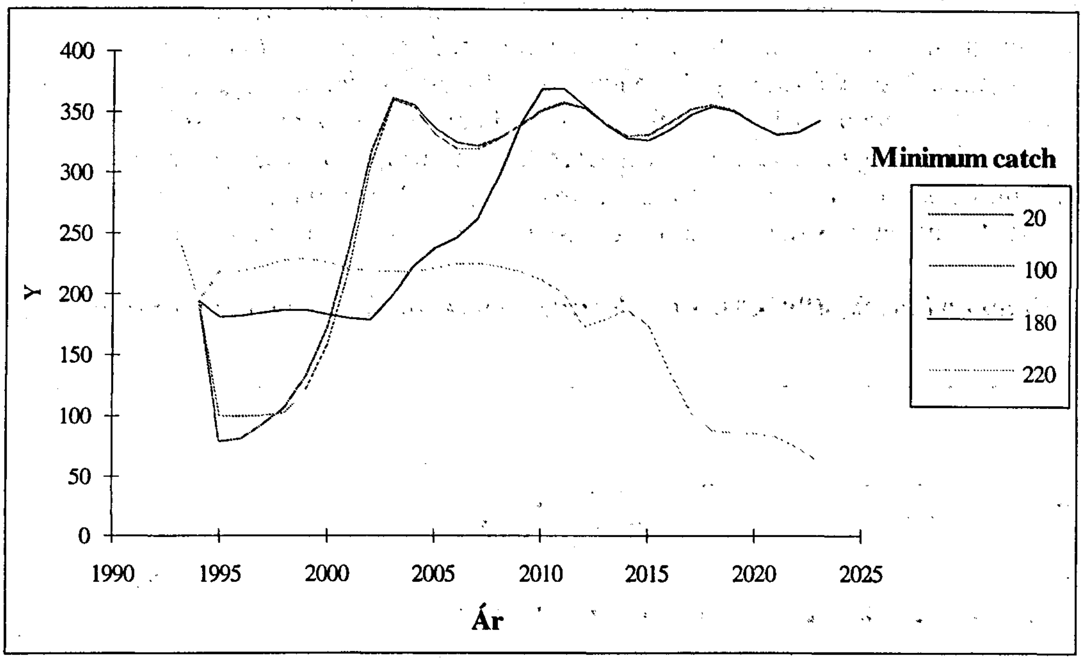
In order to obtain a feel for the model it is useful to examine its steady-state characteristics, i.e. when effort, fishing mortality, stock sizes, the catch and recruitment remain unchanged from one year to the next. As mentioned earlier, there is no fixed steady-state in the current model and hence the last 7 year-average is used.

Depending on the state of nature and the selection of *Qmin*, there may be up to 4 intersections of the steady-state catch and the catch control law (fig. 4.1). The two curves always intersect at the origin and at one value corresponding to a stable equilibrium at a level of SSB around 800 thousand tonnes. However, it is possible for the two curves also to intersect at one or two other points and this depends on the specifics of the two curves. First suppose the slope at the origin of the steady-state curve is low, so that *F=1.5* will crash the stock. Then the *Qmin* line will intersect the steady-state curve and the *F=1.5* curve, were it present in the graph, would continue to the origin, staying above the the steady-state curve. If the the steady-state curve has a higher slope at the origin so that the stock can withstand a fishing pressure corresponding to *F=1.5*, then a high *Qmin* value may cross the the steady-state curve but the *F=1.5* line will cross it again, going to the origin below the the steady-state curve.

These results are of considerable importance since they describe somewhat the nature of a potential stock collapse.

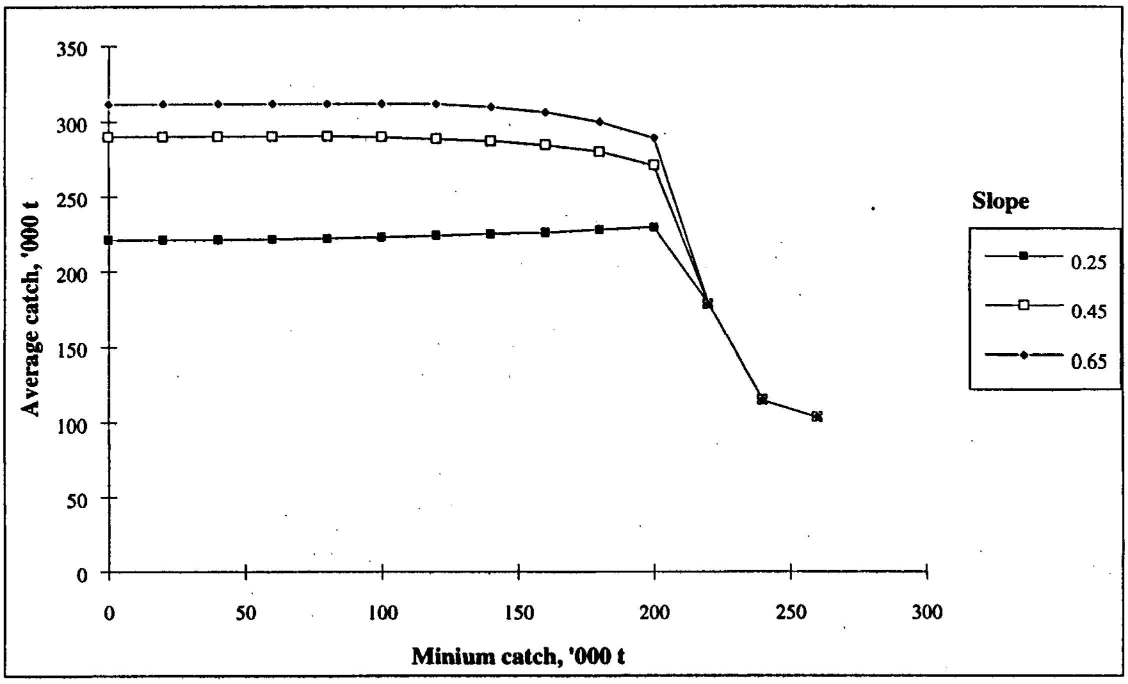
Intuitively, the likely effect of lowering minimum levels for the catch control law should be to increase the speed of recovery of the cod stock and hence to increase the catches sooner than if the minimum level is set higher. This is illustrated in fig 5.1.

Fig 5.1. Effect on catch trend of cod of different values of Qmin.



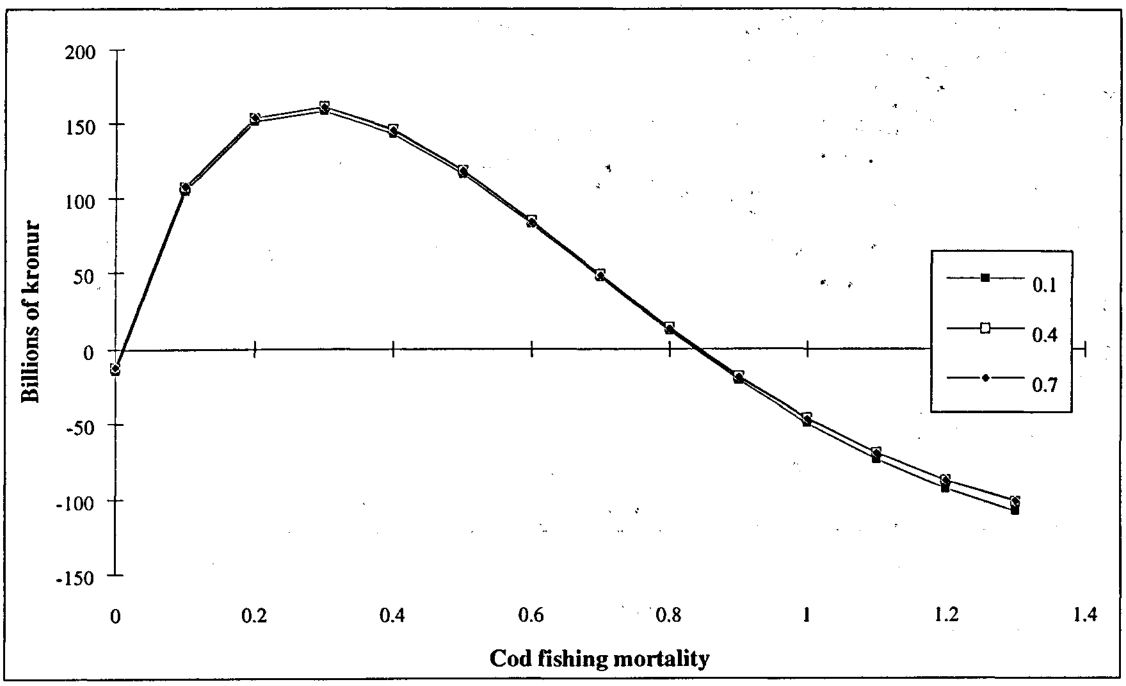
In the nonstochastic scenario one would expect that the final cod catch should increase as the minimum catch was lowered. However, since this leads to lower initial catches, it is not at all clear what the effect of lowering the minimum will have on the average annual catch from the cod stock. In general, a higher value of the slope of the catch control law would be expected to lead to a higher average catch, since the intercept is in all cases on the right hand side of the maximum in Fig 4.1. This is clear from fig 5.2, Not illustrated in this figure is the side effect of the increased slope, namely increased overall variance and interannual variability.

Fig. 5.2. Total cod catch, 1995-2023 for different values of the slope and minimum catch.



The obvious deciding issue for long-term management of the 3-species system would be the long-term profits from the fishery and the obvious control parameters would be the level of fishing from the cod and capelin stocks. The profit is defined in section 4.x and the effect of varying the levels of fishing mortality for cod and the proportion caught of capelin is given in fig 5.4. It is seen from this figure that one would expect maximum profit by fishing at a fishing mortality of about 0.3 for cod. The effect of capelin fishing at this level of cod fishing is relatively minor but it would seem that fishing on average e.g. 40% of the available capelin TAC in fall (and the remainder in spring) could be a possible guideline.

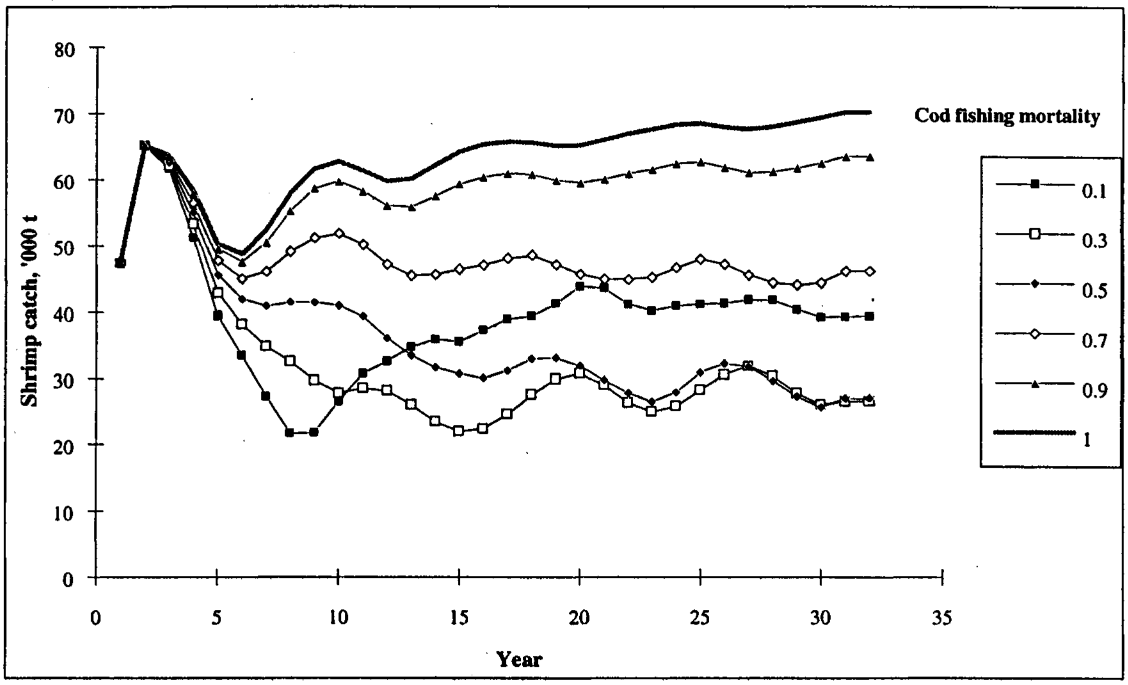
Fig 5.4 Pure profits for different levels of fishing from the cod and capelin stocks.



Further, if F=0.3 is chosen as a long-term target, then it should be noted that this target corresponds to a long-term level of SSB equal to about 800 thousand tonnes. This needs to be borne in mind when possible management strategies are developed.

One fairly important concern is the potential impact of an increase in the cod stock on the catches of Pandalus. This is illustrated in fig 5.5, where time trajectories of Pandalus catches are plotted for different levels of fishing mortality of cod. It is seen that the current model implies that the decrease in shimp catches can be quite considerable and are in fact expected to decrease from current levels of around 50 thousand tonnes to levels of 20-30 thousand tonnes if the cod stock is utilized at F=0.3.

Fig 5.5. Trends in Pandalus catches corresponding to different levels of fishing mortality of cod.



5.2 Economic results

The model can be used to define optimal policies with respect to several economic criteria. Table 5.1 shows the results from optimization using different economic criteria. The optimization was done with the annual fishing mortalities as variables. The resulting catches are shown in the first part of the table.

From economic point of view maximizing discounted profits is the obvious criteria for choosing the optimal harvesting strategy. It can though be argued that where there is widespread unemployment and overinvestment the criteria should take some account of the fact that in these situations the actual market prices may not reflect correctly the opportunity cost of using labour and capital. But these considerations will change with the general economic conditions not only in the fishing industry but in the economy as a whole. It therefore seems reasonable to use the sum of discounted profits as a criteria to decide on the long run stock level and catches but to choose an adjustment path which takes some account of the specific economic conditions in the country during some adjustment period. In the calculations presented in table 5.1 the length of the adjustment period was set at 11 years, starting in 1995 and ending in 2005. In the calculation the recruitment of capelin, was assumed constant and set at the estimated average. Throughout the calculations a 5% discount rate was used.

Table 5.1 Optimization results

	(1)	(2)	(3)	(4)	(5)
	Cod catch, '000 tonnes				
1993	251	251	251	251	251
1994	190	190	190	190	190
1995	0	0	0	82	84
1996	0	0	95	106	106
1997	36	75	164	133	133
1998	176	193	190	166	165
1999	253	260	219	199	199
2000	278	287	249	231	230
2001	299	307	274	255	255
2002	322	326	290	271	270
2003	333	337	299	281	281
2004	330	336	307	292	291
2005	345	356	356	356	356
2023	347	356	356	356	356
	Cod SSB, '000 tonnes				
1994	223	223	223	223	223
2023	835	763	762	762	762
	Exploitable biomass, '000 tonnes				
1994	346	346	346	346	346
2023	1473	1378	1378	1378	1378
	Fishable biomass, '000 tonnes				
1994	593	593	593	593	593
2023	1668	1612	1612	1612	1612
	Present value of profits from fishing and processing of cod. Billions of IKR				
1994	201	200	196	192	191
2023	258	255	255	255	255
	Present value of profits from fishing and processing of cod, capelin and shrimp. Billions of IKR				
1994	194	195	193	189	189
2023	246	245	245	245	245
	Capelin catches, '000 tonnes				
1994	896	896	896	896	896
2023	323	385	385	385	385
	Shrimp catches, '000 tonnes				
1994	64	64	64	64	64
2023	29	28	28	28	28
	Effort in fishing for cod (index)				
1994	100	100	100	100	100
2023	66	71	71	71	71

(1):The sum of discounted profits from fishing and processing of cod is maximized.

(2): The sum of discounted profits from fishing and processing of cod, capelin and shrimp is maximized.

(3): Adjustment to profit-maximizing stock and catch levels during 11 years.
The path of adjustment decided by maximizing the discounted value added.

(4): As in (3), but the adjustment path decided by maximizing discounted utility with moderate risk aversion (s=2).

(5): As in (3), but the adjustment path decided by maximizing discounted utility with high risk aversion (s=6).

The second section of table 5.1 shows the size of the spawning stock of cod. The figure for 2023, the last year in the calculations shows the optimal size. The third section of the table shows the exploitable biomass and the fourth shows the fishable biomass. The fifth section shows the present value of profits from fishing and processing of cod given the different harvesting strategies. These figures are sensible estimates of the economic value of the resource. The calculations show that harvesting strategies which lead to a recovery of the cod stock to its optimal size will increase the value of the resource by some 60 billions of Icelandic kronors (IKR) to some 255 billions IKR or 3.6 billions USD. These figures can be compared to the annual GDP in Iceland of almost 400 billions IKR or 5.7 billions USD.

The sixth section shows the present value of profits from fishing and processing of cod, capelin and shrimp. It is worth noting that these figures are smaller than those for cod alone. The reason for this is that the capelin industry is expected to operate at a loss when the cod stock increases and the capelin catches decrease. This conclusion is though very much dependent on the adjustment of the investment in factories and vessels to the development of the capelin catches and of alternative uses for these factors of production. E.g. if catches of herring will increase this will cause better utilisation of the capital in the capelin industry and change the economic situation of this industry quite dramatically. Given all these uncertainities concerning the capelin industry and the enormous variability of the capelin stock itself it is natural to be cautious in drawing conclusions from simple models. It is though to be expected that when one considers a model with more than one specie it may be optimal to decrease the exploitation of one specie to allow for increased exploitation of another specie which is more profitable. Sections seven and eight show the estimated decrease in the exploitation of capelin and shrimp as the cod stock is allowed to increase.

In the calculations presented in table 5.1 unemployment of labour and capital is taken into account by assuming the cost of these factors of production ot be zero (column 3-5). This assumption is obviously extreme. Yet in all cases the calculations indicate that it is highly beneficial to reduce fishing immediately and drastically. The calculations give a long-term value of SSB in the range 750-850 thousand tonnes. This value is a useful aim for management policies which take a long-term view.This can be used to form a basis for a management procedure.

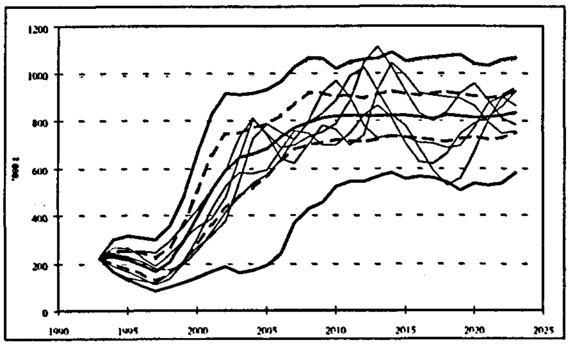
6. Simulation results

It should be pointed out that the management procedure was determined so that it would yield a long-term average spawning stock of about 800 thousand tons. This corresponds to the pre-selection of an equilibrium spawning stock which is be acceptable from both an economic and a biological standpoint.

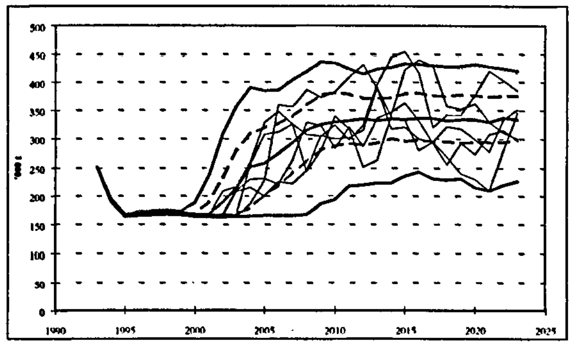
Presented in this section are detailed results from a management procedure where attempts are made to maintain catches above 165 thousand tonnes. Summaries are given of results for alternate management procedures and alternate assumptions on the biology of the species involved.

Fig. 6.1 Simulation results. SSB-based CCL and annual minimum cod catch of 165 000 t.

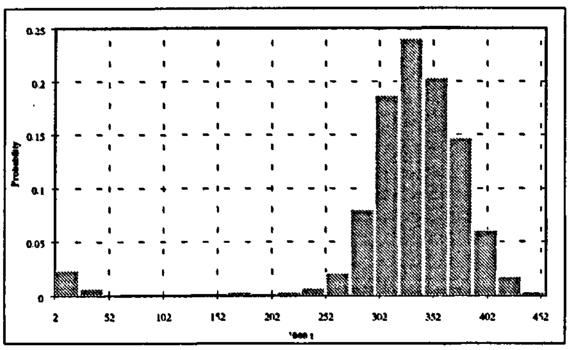
Spawning stock biomass



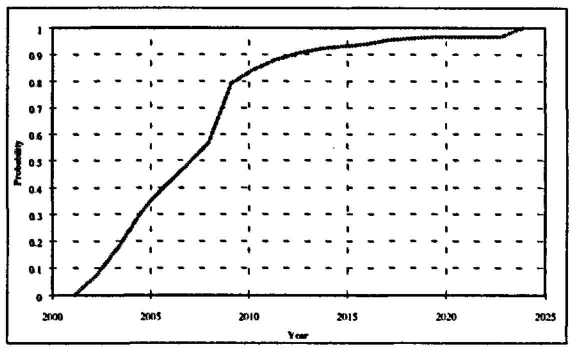
Cod catch



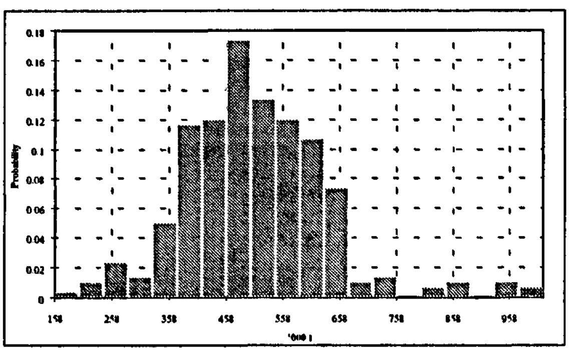
Histogram of average cod catch 2017-2023



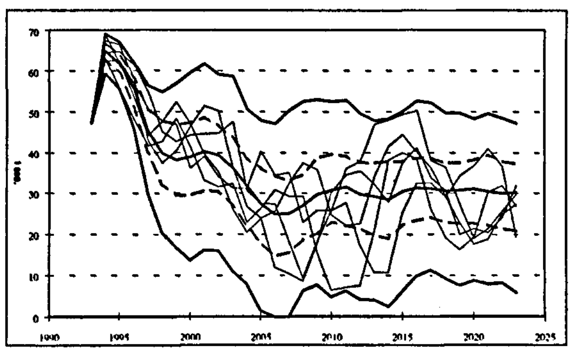
Cumulative probability distribution of first year with cod catch over 300.000 tonn



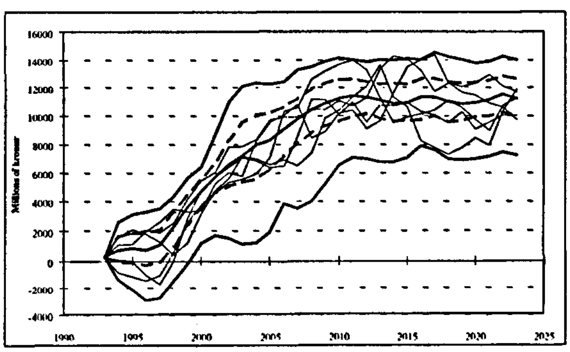
Histogram of av. capelin catch 2017-2023



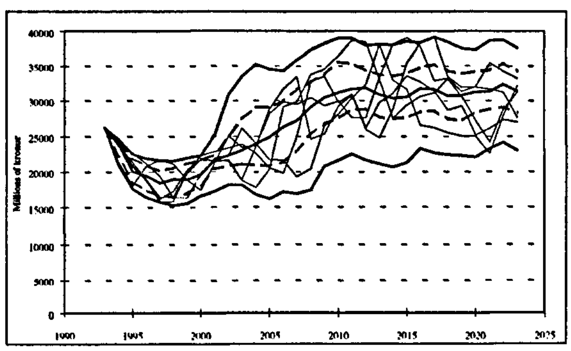
Shrimp catch



Total profits



Contribution to GNP



Figures 6.1 shows the basic results from a simulation using the SSB-based management procedure with a minimum catch limit set to 165 thousand tonnes for cod. The 5 line diagrams each contain several different line types. The thin lines show the simulated trends for 5 sample simulations. The central solid thick line denotes the median value from the 300 simulations conducted. The top and bottom solid thick lines indicate the lower and upper 5% of the distribution of results and finally, the dashed thick lines depict the upper and lower 25% of the results.Thus, 5 percent of the stock sizes each year lie below the lowest line and 5 percent lie above the highest one and, similarly, 25 percent of the stock sizes each year lie below the lower broken line and 25 percent lie above the higher one.

For example, the top left panel shows results concerning the cod SSB from 300 simulations. It is seen that in the year 1997, there is a 50% chance that the stock will still be below 200 thousand tonnes. It is also seen that there is less than a 5% chance that the stock will decline towards zero (the estimated probability is 3% as may be inferred from the histogram second from the top in the left column).

Numerical summaries of economic results from the simulations are given in table 6.1. Deviations are considered in section 7.

Table 6.1. Economic summaries for the base case

Minimum catch set to 165.000 tonnes. Net present value using 5% discount rate.
Billions of kronur

		Gross revenues	*Pure profit*	*GDP contrib.*	*Green GDP contrib.*
Cumul. probability					
	5%	717	98	487	37
	25%	749	132	516	41
	50%	767	150	535	43
	75%	782	161	548	45
	95%	806	179	570	48
Mean		760	141	526	41
Std. deviation		44	46	47	11

Cod		*Gross revenues*	*Pure profit*	*GDP contrib.*	*Green GDP contrib.*
Cumul. probability					
	5%	506	96	366	27
	25%	552	132	410	33
	50%	585	153	438	36
	75%	605	166	456	39
	95%	639	187	487	42
Mean		571	143	425	34
Std. deviation		72	51	67	13

Capelin		*Gross revenues*	*Pure profit*	*GDP contrib.*
Cumul. probability				
	5%	71	-18	21
	25%	79	-16	27
	50%	84	-15	30
	75%	90	-13	35
	95%	99	-11	41
Mean		85	-14	31
Std. deviation		10	3	7

Shrimp		*Gross revenues*	*Pure profit*	*GDP contrib.*	*Green GDP contrib.*
Cumul. probability					
	5%	84	9	56	4
	25%	94	11	63	6
	50%	101	12	68	7
	75%	108	13	72	8
	95%	122	15	82	10
Mean		104	12	70	8
Std. deviation		21	3	14	3

7. Further developments and tests

The biological simulation model described in the previous sections includes several assumptions. It is of interest to consider the effect of changing some of these assumptions or including some missing sources of variation. This section considers these issues by examining some of the effects of individual changes to the base model.

The deviations from the base computations are of two types: Changes to the catch control law and changes to the biological assumptions. Results from the analysis of several deviations are given in table 7.1.

The overall conclusions from this table is that the only deviation which has a major effect is a change in the minimun catch limit. Increasing the lower limit set on catches to 190 thousand tonnes will increase the collapse probability to 20% and decrease all expected benefits accordingly. Similarly, reducing the initial catch to 140 thousand tonnes is expected to give higher gains (column J) and safer utilisation (column A) of the resource. It is also worthy of note that greater "smoothing" of catches seems to have little effect on the results other than to reduce the amplitude of inter-annual fluctuations in catches. Also, the fishable biomass rule seems more or less equivalent to the SSB-based one in most respects.

Table 7.1 Sensitivity analysis.

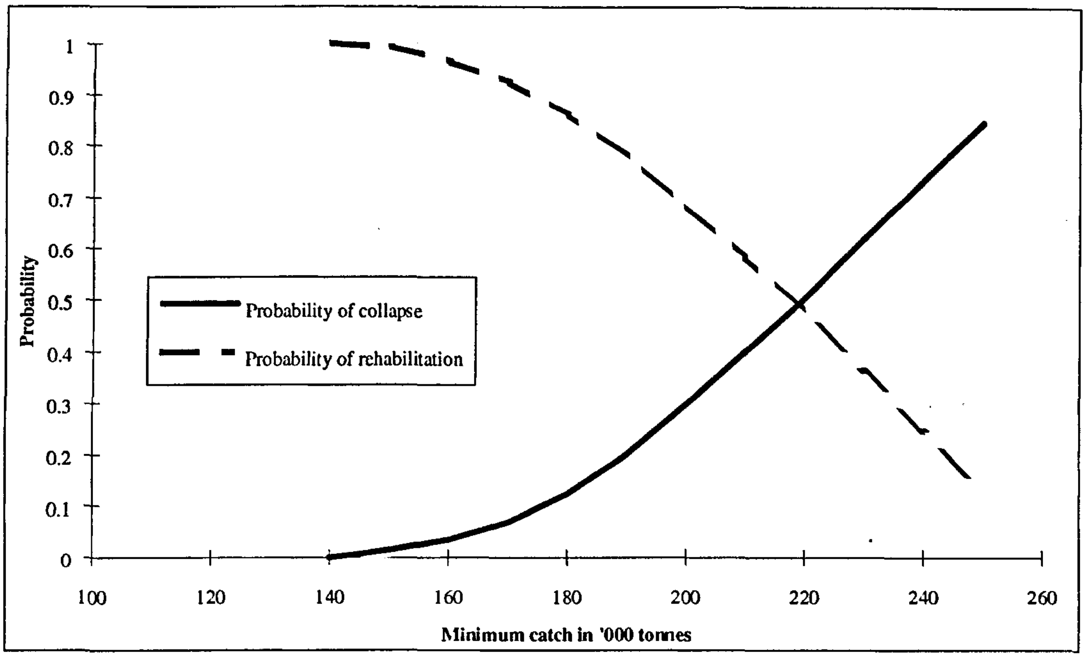
	See explanations below												
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*
Base case (Qmin=165)	3.0%	2000	96.7%	2007	806	328	30	509	31	141	46	526	47
CCL deviations													
Qmin=190,000 tonnes	20.0%	2000	77.7%	2008	664	275	26	588	40	93	95	489	94
Qmin=140,000 tonnes	0.0%		100.0%	2005	834	340	31	494	29	155	17	534	21
Previous year weight=0,25	3.0%	2000	96.7%	2006	805	329	45	509	31	141	46	527	47
Previous year weight=0,75	3.0%	2000	96.7%	2008	801	328	16	512	31	140	46	521	45
Fishable biomass rule	3.0%	2001	96.7%	2007	763	332	28	540	30	139	45	535	48
Biological deviations													
Growth independent of dens.	5.0%	2002	94.3%	2007	797	333	26	517	25	139	56	523	56
Recruitment 10% higher	2.0%	2001	97.3%	2006	863	348	30	469	28	152	40	541	41
Recruitment 10% lower	4.7%	2000	95.3%	2008	742	305	29	552	34	128	52	508	53
Cyclic recruitment	2.3%	2004	97.3%	2006	851	336	34	481	29	145	40	532	42
Indep. errors in stock est.	1.4%	2003	99.0%	2007	813	334	32	503	30	148	32	533	33

Legend

A	Probability that the cod stock is below 100.000 tonnes in 2024
B	Average year when the cod stock drops below 100.000 tonnes, in those simulations when this occurs
C	Probability that the cod catch exceeds 300.000 tonnes before 2024
D	Average year when the cod catch exceeds 300.000 tonnes, in those simulations when this ocurs
E	Average cod SSB 2017-2023, '000 tonnes
F	Average cod catch 2017-2023, '000 tonnes
G	Average interannual changes in cod catch 2017-2023, '000 tonnes
H	Average capelin catch 2017-2023, '000 tonnes
I	Average shrimp catch 2017-2023, '000 tonnes
J	Expected present value of profits, billions of kronur
K	Standard deviation of present value of profits, billions of kronur
L	Expected present value of GNP, billions of kronur
M	Standard deviation of present value of GNP billions of kronur

As the probability of collapse is obviously an extremely important characteristic of any catch rule it was decided to estimate it separately as a function of the minimum catch. The results are displayed in Figure 7.1. Also displayed is the probability of catches exceeding 300.000 tonnes in the simulation period (i.e. before 2023). That event is termed "rehabilitation" in the Figure. The two curves are almost mirror images of one another - the probability of collapse rising from zero at a minimum catch of 140.000 tonnes to 50% at a minimum catch of 220.000 tonnes and the probability of rehabilitation falling from 100% to 50% over the same range.

Figure 7.1 Probabilities of collapse and rehabilitation



8. Discussion

The results indicate that overall gains from fishing for the three species considered in this paper can be enhanced considerably by an initial decrease in fishing for cod, which would allow the cod stock to recover. A long-term aim of maintaining the SSB at about 800 thousand tonnes or F around 0.3 would be suitable for this purpose. Several means exist to approach such a goal and in particular it is seen that reducing initial catches of cod to about 165 thousand tonnes would allow the cod stock to approach such a value with quite high probability. It is clear, however, that this result is quite sensitive to input values and minor changes in the baseline assessment can change this probability estimate considerably and intital reductions to below 150 thousand tonnes would be desirable from a purely economic point of view. It follows that the use of the catch control law considered but without an initial minimum catch would be a useful means towards the goal of a profitable and sustainable fishery.

These results are qualitatively similar to those obtained for the cod stock as described in Baldursson *et al.* (1993), even though only a single species model was considered in that paper. In the present paper the reduction in capelin and shrimp catches due to an increase in the cod stock is taken into account as well as the potential effect of food shortage on the growth of cod. It is seen that these factors only marginally change the qualitative results for the cod stock alone although the estimate of the absolute level of catches obtainable from the system changes somewhat.

Most of the models used in this paper are based on relationships derived to at least some extent from available data. In some cases models have been used without much data support and it is clear that further analyses of existing data sets should be conducted in order to ascertain some of the assumed relationships. For example, the model for changes in the natural mortality of juvenile capelin has not been verified although this might be possible by careful analysis of existing data on acoustic abundance of capelin and VPA abundance of cod. Analyses have indicated, however, that overall consumption of capelin by cod is considerable and hence this effect needs to be accounted for.

Earlier modelling work has also indicated that cod may switch from capelin as prey to other food items in years of low capelin abundance. This is not modelled in the present paper. This will have several consequences and further research needs to be done on this topic. It should be noted that effects of such switching may be manyfold. Firstly, cod may switch to shrimp and in this case further catch reductions will be inflicted on the shrimp fishery. This particular effect is unlikely to alter the basic conclusions to any significant extent, since in the present version, shrimp catches are cut by almost 50% and in spite of this, the rebuild of the cod stock is well worth while. Another consequence of low capelin abundance is that cod may now need more food than is available. Since explicit food balancing is not done in the model, this is beyond its scope. Therefore, the precise effect of low capelin abundance resulting in cod switching to lower-preference prey such as shrimp may have the effect that the shrimp stock becomes "overexploited" by cod with the net effect that the cod mean weights at age go down. This is partly accounted for in the density-dependent weight reduction in the cod stock, which does incorporate the type of weight reduction that has been seen in historical data sets. Future work should be done to model the effect explicitly in order to illiustrate the potential effect. Such a model should not use the present model for mean weights at age but rather a model where mean weight at age is related to biomass of capelin per unit abundance of cod *etc.*

Appendix A. Some numerical details

In order to apply a catch-control law a method is required to change a quota in weight to a fishing mortality. This is usually done by writing the yield in tonnes as a function of the overall fishing mortality, F:

$$Y(F) = \sum_a \frac{w_a F s_a}{F s_a + M_a}\left(1 - e^{-(F s_a + M_a)}\right) N_a$$

and then solving F from the equation $Y(F) = Q$, where Q is the quota in tonnes prescribed by the procedure. Clearly this equation is nonlinear in F and a direct solution does not exist. Since a spreadsheet was used for the simulations described in this paper, an iterative procedure was not considered acceptable and hence the following solution was developed.

A third-degree Taylor-approximation to $e^{-(F s_a + M)}$ yields the equation

$$Q \approx \sum_a \frac{w_a F s_a}{F s_a + M_a}\left(1 - \left\{1 - (F s_a + M_a) + (F s_a + M_a)^2 / 2 - (F s_a + M_a)^3 / 3\right\}\right) N_a$$

$$= \sum_a w_a F s_a \left(\left\{1 - (F s_a + M_a)/2 + (F s_a + M_a)^2 / 3\right\}\right) N_a$$

where terms can be collected into a third-degree equation in F which can be solved analytically with a single real solution in all cases considered.

In this fashion F can be computed directly from Q which is determined from the size of the spawning stock biomass in the previous year. Hence no iteration is required to find a fishing mortality corresponding to a quota.

Upon testing, it is found that the solution to the cubic equation yields an estimate that tends to give catches that are a few percent to low, typically about 5 thousand tonnes for the Icelandic cod, where the total catch is typically between 200 and 350 thousand tonnes. Although it may be feasible to find better approximations and, indeed these discrepancies are minor, the simple solution taken was to lower the target Q-value within the equation-solving procedure. This approach gave fishing mortalitites corresponding to Q-values usually within 2 thousand tonnes of the target.

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